

# Robot Structural Analysis Professional



## Comparison with AFNOR Benchmarks

*“Guide de validation des progiciels de calcul de structures”*  
**AFNOR, 1990**

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## INTRODUCTION

This verification manual contains a range of static and dynamic benchmark tests covering fundamental types of behaviour encountered in structural analysis. 58 examples of static, dynamic, and thermo-mechanics problems are solved using bar, plate, and shell FE. All the examples have been taken from:  
***"Guide de validation des progiciels de calcul de structures" AFNOR, 1990***

Benchmark results (signed as "AFNOR") are recalled, and compared with results of **ROBOT Structural Analysis Professional 2025**.

Each problem contains the following parts:

- the name of the benchmark as used in the AFNOR guide,
- short problem description,
- scheme of the model,
- comparison between ROBOT Structural Analysis Professional results and reference values.

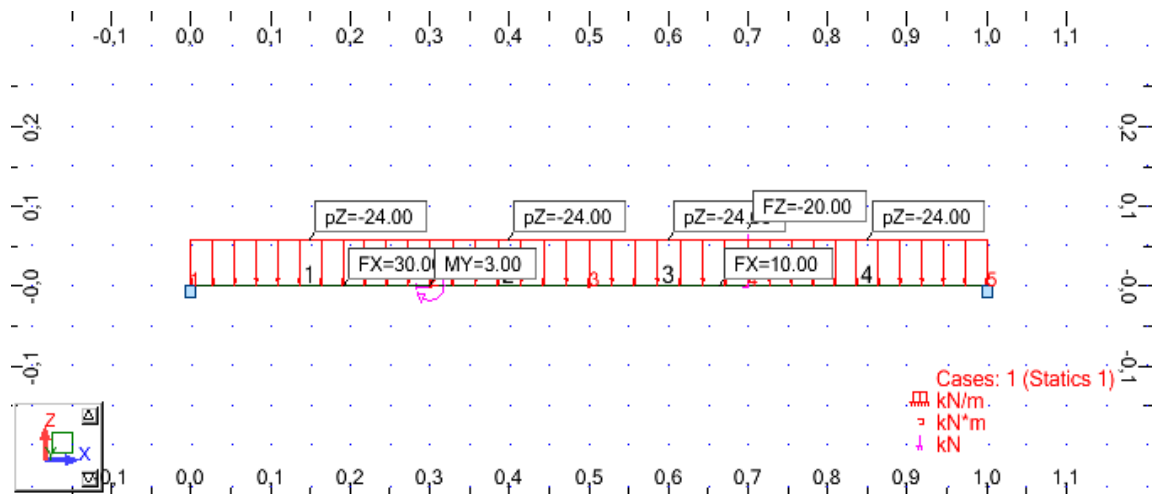
# STATIC ANALYSIS

## 1. BAR STRUCTURES

## VERIFICATION EXAMPLE

### 2D Euler's beam bending - SSSL01

|                          |  |        |
|--------------------------|--|--------|
| <b>Name of the test:</b> |  | SSLL01 |
| <b>Reference:</b>        |  | AFNOR  |
| <b>Specification:</b>    | Uni-directional bending of an elastic (Euler) beam |        |
| <b>GEOMETRY:</b>         | Length: L = 1,0 m, fixed ends                      |        |
|                          | Section: $I_y=1.7e-8$ , $E=2.1e11$ [N, m]          |        |



**DATA FILE:** SSSL01.rtd

#### COMPARISON:

| Node | Compared result           | Value                     |                                 | Difference % |
|------|---------------------------|---------------------------|---------------------------------|--------------|
|      |                           | RSA<br>(ROBOT<br>results) | AFNOR<br>(Referenced<br>values) |              |
| 3    | Shearing force (N)        | 540                       | -540                            | 0.0          |
| 3    | Bending moment (Nm)       | 2800                      | 2800                            | 0.0          |
| 3    | Vertical displacement (m) | -4.90196e-2               | -4.90196e-2                     | 0.0          |
| 1    | Horizontal reaction (N)   | -24000                    | -24000                          | 0.0          |

#### CONCLUSIONS:

Exact agreement of results.

The different signs of shear forces arise from different local coordinates sign convention.

## VERIFICATION EXAMPLE

### 2D Timoshenko's beam bending - SSSL02

**Name of the test:**

SSLL02

**Reference:**

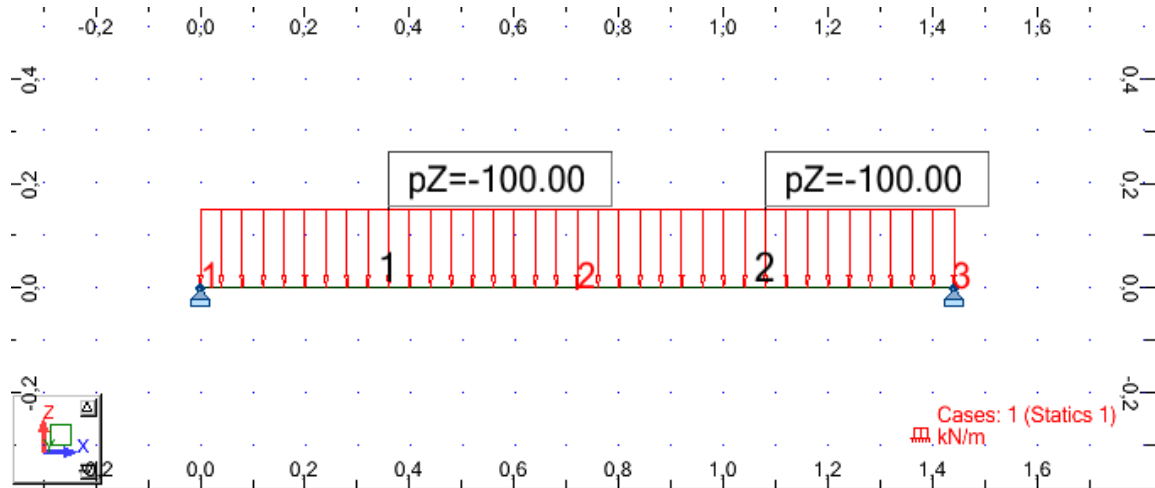
AFNOR

**Specification:**

Influence of shearing stresses (Timoshenko's beam)  
Elastic, linear, isotropic material

**GEOMETRY:**

Length:  $L = 1,44$  m, simply supported ends  
Section:  $I_y=2810e-8$ ,  $A_x=31e-4$ ,  $A_z=(31E-4/2.42)$ ,  $E=2.0e11$ ,  $\nu=0.3$



**DATA FILE:**

SSLL02.rtd

**COMPARISON:**

| Node | Compared result  | Value       |             | Difference % |
|------|------------------|-------------|-------------|--------------|
|      |                  | RSA         | AFNOR       |              |
| 2    | displacement (m) | -1.25926e-3 | -1.25926e-3 | 0.0          |

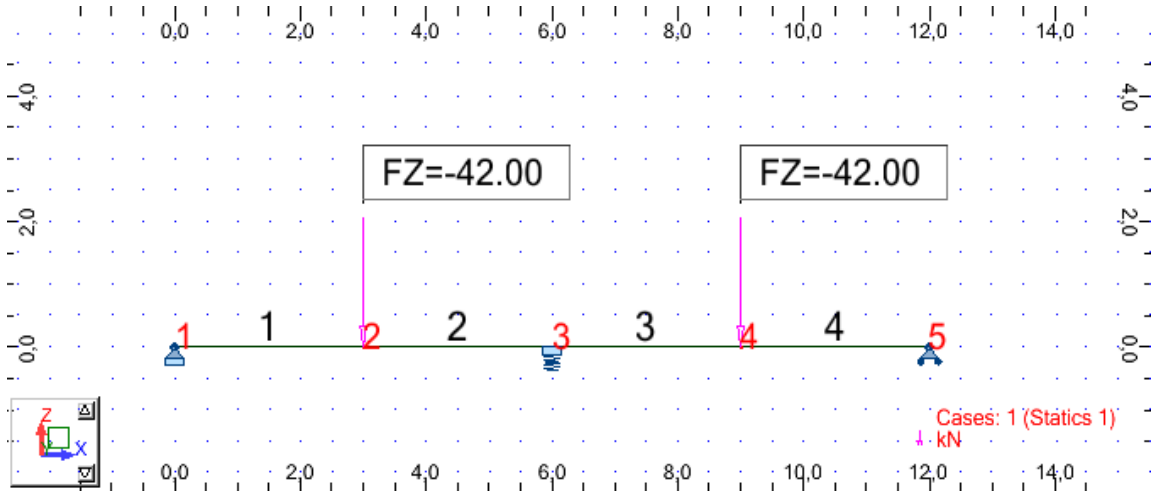
**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### Beam with elastic support - SSL03

**Name of the test:** SSL03  
**Reference:** AFNOR  
**Specification:** Simple beam under bending with elastic support in the centre of length; material: elastic, linear, isotropic.  
**GEOMETRY:** Length: L = 12 m, simply supported at ends and in the middle  
 Section:  $I_y=6.3 \text{ e-4}$ ,  $E=2.1\text{e}11$   
 Stiffness  $K_z=2.1 \text{ e6 N/m}$ .



**DATA FILE:** SSL03.rtd

**COMPARISON:**

| Node | Compared result       | Value  |        | Difference % |
|------|-----------------------|--------|--------|--------------|
|      |                       | RSA    | AFNOR  |              |
| 3    | Bending moment (Nm)   | 63000  | 63000  | 0.0          |
| 3    | Displacement UZ (m)   | -0.010 | -0.010 | 0.0          |
| 3    | Vertical reaction (N) | 21000  | 21000  | 0.0          |

**CONCLUSION:**

Exact agreement of results.



## VERIFICATION EXAMPLE

### 3D frame with elastic supports - SSSL04

**Name of the test:**

SSLL04

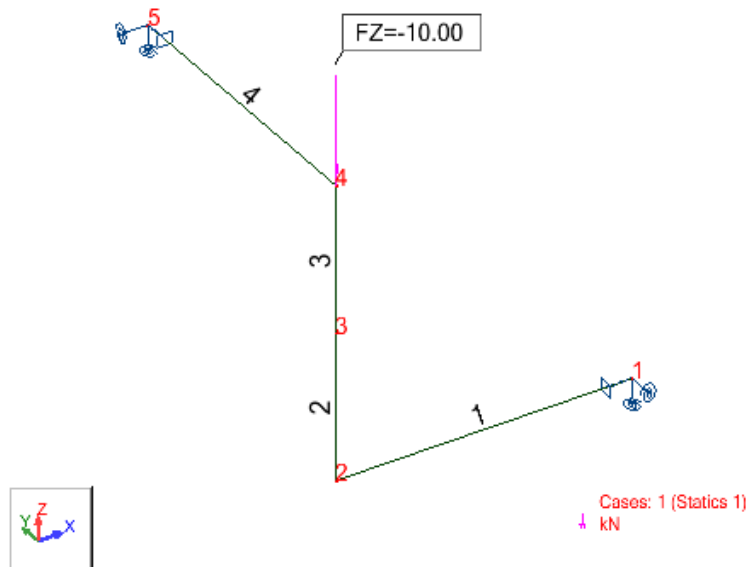
**Reference:**

AFNOR

**Specification:**

Spatial frame with elastic supports, under bending and torsion; material: elastic, linear, isotropic (non-compressible bars assumed)

**GEOMETRY:**



**DATA FILE:**

SSLL04.rtd

**COMPARISON:**

| Node | Compared result     | RSA       | AFNOR     | Difference % |
|------|---------------------|-----------|-----------|--------------|
| 5    | Moment MX (Nm)      | 1562.5    | 1562.5    | 0.0          |
| 5    | Moment MY (Nm)      | -8437.5   | -8437.5   | 0.0          |
| 5    | Moment MZ (Nm)      | 3125.0    | 3125.0    | 0.0          |
| 1    | Moment MX (Nm)      | -1562.5   | -1562.5   | 0.0          |
| 1    | Moment MY (Nm)      | -8437.5   | -8437.5   | 0.0          |
| 1    | Moment MZ (Nm)      | 3125.0    | 3125.0    | 0.0          |
| 5    | Displacement UY (m) | -0.029762 | -0.029762 | 0.0          |
| 5    | Rotation RX (rad)   | 0.16071   | 0.16071   | 0.0          |
| 4    | Displacement UZ (m) | -0.37004  | -0.37004  | 0.0          |

**CONCLUSIONS:**

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Exact agreement of results.

## VERIFICATION EXAMPLE

### Bending of rigidly connected beams - SSSL05

**Name of the test:**

SSSL05

**Reference:**

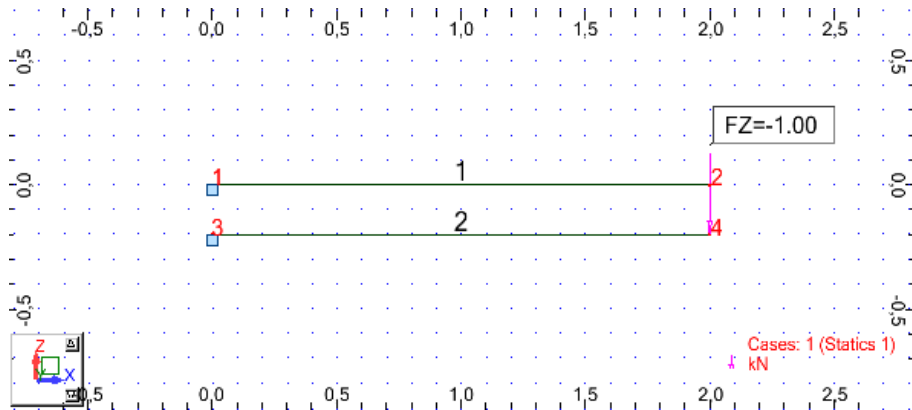
AFNOR

**Specification:**

Beams with rigid link – bending of non-compressible bars

**GEOMETRY:**

Length:  $L = 2$  m, distance 0,2 m,  
Left ends - fixed, right – rigidly linked  
Section:  $I_z = 4/3e-8$ ,  $A_x = 1.0$ ,  $E = 2e11$



**DATA FILE:**

SSSL05.rtd

**COMPARISON:**

| Node | Compared result       | RSA    | AFNOR  | Difference % |
|------|-----------------------|--------|--------|--------------|
| 2    | Displacement (m)      | -0.125 | -0.125 | 0.0          |
| 4    | Displacement (m)      | -0.125 | -0.125 | 0.0          |
| 1    | Vertical reaction (N) | 500    | 500    | 0.0          |
| 1    | Moment My (Nm)        | -500   | 500    | 0.0          |
| 3    | Vertical reaction (N) | -500   | 500    | 0.0          |
| 3    | Moment My (Nm)        | -500   | 500    | 0.0          |

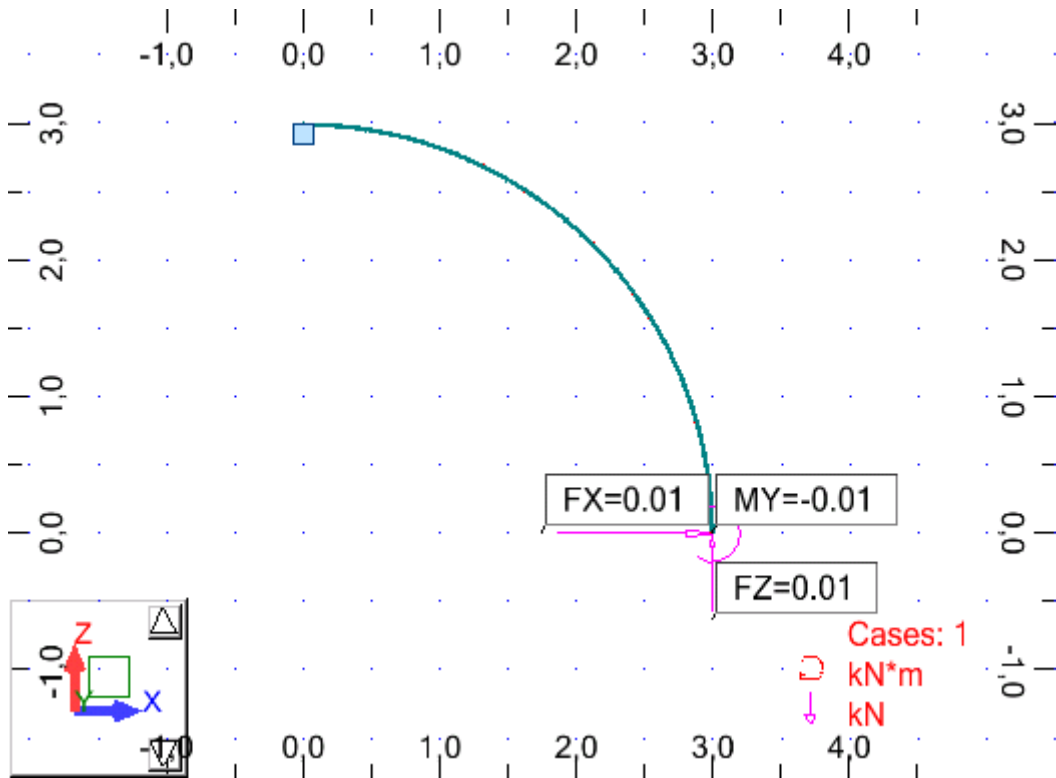
**CONCLUSION:**

Exact agreement of results  
(taking into account different sign convention).

## VERIFICATION EXAMPLE

### 2D circular arch bending - SSSL06

|                          |   |        |
|--------------------------|---|--------|
| <b>Name of the test:</b> |   | SSLL06 |
| <b>Reference:</b>        |   | AFNOR  |
| <b>Specification:</b>    | Bending of a quarter circle, fixed at the end, made from hollow round section; elastic linear material, non-compressible bars assumed |        |
| <b>GEOMETRY:</b>         | Radius of the circle = 3 m, upper end fixed.<br>Section: R=10mm, thickness=2mm (Ax=1.0), E=2e11                                       |        |



**DATA FILE:** SSSL06.rtd

**COMPARISON:**

| Node | Compared result    | RSA    | AFNOR  | Difference % |
|------|--------------------|--------|--------|--------------|
| 91   | Displacement UX(m) | 0.3791 | 0.3791 | 0.0          |
| 91   | Displacement UZ(m) | 0.2417 | 0.2417 | 0.0          |
| 91   | Rotation RY (rad)  | 0.1654 | 0.1654 | 0.0          |

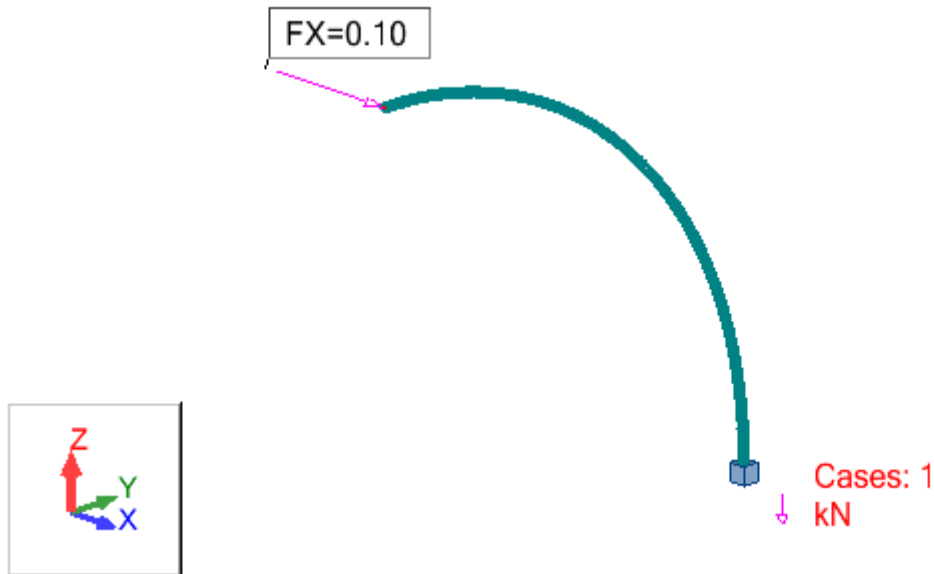
**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### 3D circular arch transverse bending - SSSL07

|                          |  |        |
|--------------------------|--|--------|
| <b>Name of the test:</b> |  | SSLL07 |
| <b>Reference:</b>        |  | AFNOR  |
| <b>Specification:</b>    | A quarter circle fixed at the end, bending and torsion of a thin-walled (hollow) round section; material elastic linear isotropic. |        |
| <b>GEOMETRY:</b>         | Radius of the circle = 1 m, lower end fixed.<br>Section: R=10mm, thickness=2mm, E=2e11   |        |



**DATA FILE:** SSSL07.rtd

#### COMPARISON:

| Node | Compared result     | RSA           | AFNOR   | Difference % |
|------|---------------------|---------------|---------|--------------|
| 91   | Displacement UX (m) | 0.13461       | 0.13462 | 0.0          |
| 16   | Moment MX (Nm)      | 74.115 (mean) | 74.118  | 0.04         |
| 16   | Moment MZ (Nm)      | 96,589 (mean) | 96.592  | 0.03         |

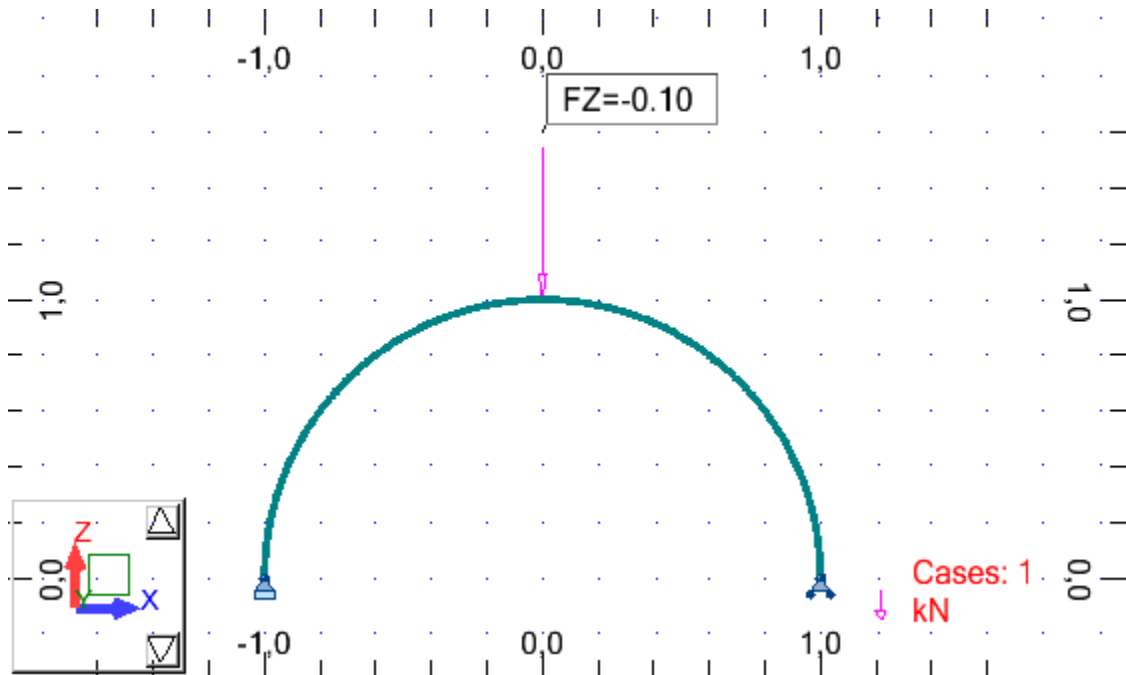
#### CONCLUSIONS:

Results correct.

## VERIFICATION EXAMPLE

### 2D semi-circular arch bending - SSSL08

|                          |   |
|--------------------------|---|
| <b>Name of the test:</b> | SSLL08  |
| <b>Reference:</b>        | AFNOR   |
| <b>Specification:</b>    | A half circle simply supported at the ends, bending of a thin-walled (hollow) round section; material elastic linear isotropic. |
| <b>GEOMETRY:</b>         | Radius of the circle = 3 m, upper end fixed.<br>Section: R=10mm, thickness=2mm (Ax=1.0), E=2e11                                 |



**DATA FILE:** SSSL08.rtd

#### COMPARISON:

| Node | Compared result     | RSA        | AFNOR      | Difference % |
|------|---------------------|------------|------------|--------------|
| 1    | Rotation RY(rad)    | 3.0775e-2  | -3.0774e-2 | 0.0          |
| 181  | Rotation RY(rad)    | -3.0774e-2 | 3.0774e-2  | 0.0          |
| 91   | Displacement UZ (m) | -1.9206e-2 | -1.9206e-2 | 0.0          |
| 181  | Displacement UX (m) | 5.3911e-2  | 5.3912e-2  | 0.0          |

#### CONCLUSIONS:

Exact agreement of results.

## VERIFICATION EXAMPLE

### Plane truss with nodal loads - SSSL09

**Name of the test:**

SSL09

**Reference:**

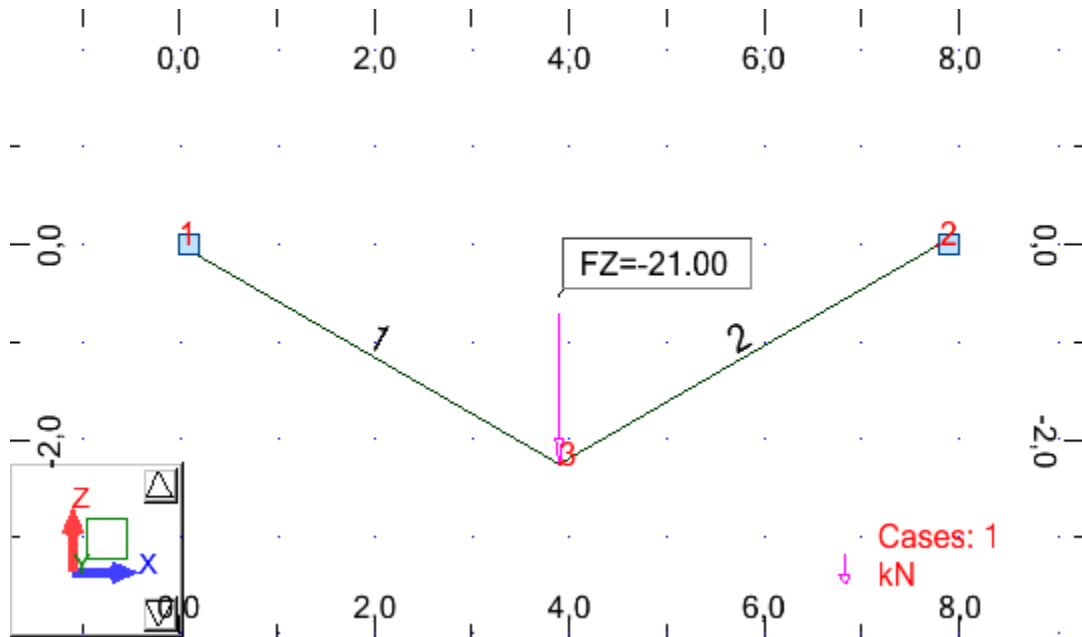
AFNOR

**Specification:**

Truss made of two bars; material: elastic, linear, isotropic.

**GEOMETRY:**

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**DATA FILE:**

SSL09.rtd

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**COMPARISON:**

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| Node  | Compared result     | RSA        | AFNOR      | Difference % |
|-------|---------------------|------------|------------|--------------|
| 3     | Displacement UZ (m) | -3.000 e-3 | -3.000 e-3 | 0.0          |
| 1 - 3 | Tensile force (N)   | 21.000 e+3 | 21.000 e+3 | 0.0          |
| 2 - 3 | Tensile force (N)   | 21.000 e+3 | 21.000 e+3 | 0.0          |

**CONCLUSION:**

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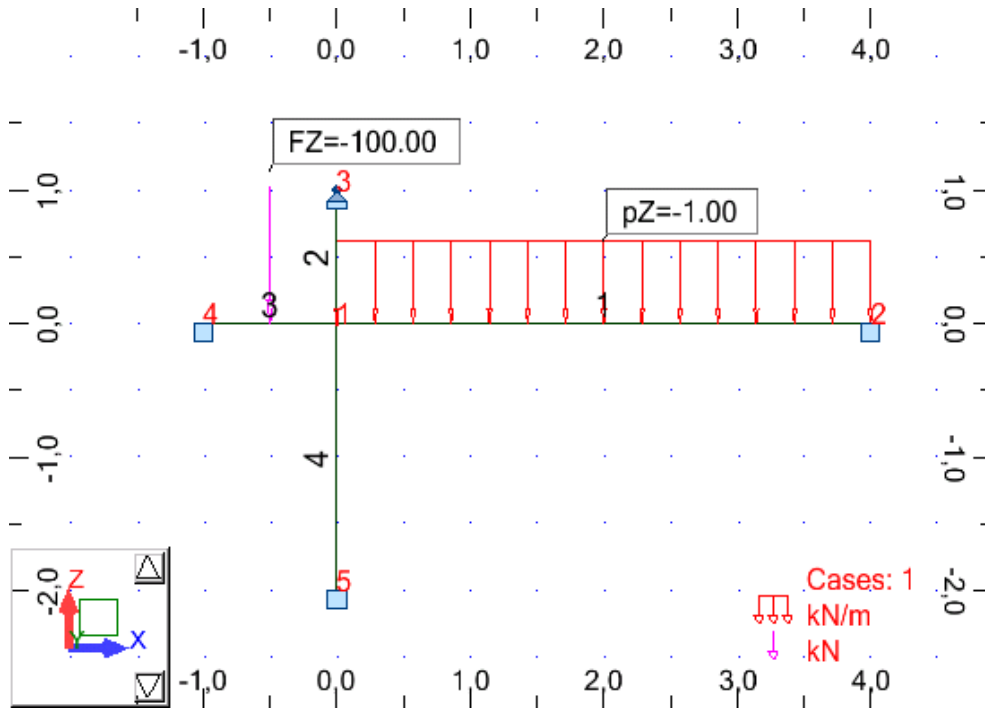
Exact agreement of results.

## VERIFICATION EXAMPLE

### Plane frame with uniform loads - SSSL10

**Name of the test:** SSSL10  
**Reference:** AFNOR  
**Specification:** Frame made of four bars with different moments of inertia; material: elastic, linear, isotropic (non-compressible bars assumed)

**GEOMETRY:**



**DATA FILE:** SSSL10.rtd

**COMPARISON:**

| Node  | Compared result   | RSA        | AFNOR      | Difference % |
|-------|-------------------|------------|------------|--------------|
| 1     | Rotation RY (rad) | -0.227119  | 0.227118   | 0.0          |
| 1 - 1 | Moment MY (Nm)    | -11023.72  | 11023.72   | 0.0          |
| 2 - 1 | Moment MY (Nm)    | -113.559   | 113.559    | 0.0          |
| 3 - 1 | Moment MY (Nm)    | -12348.59  | -12348.588 | 0.0          |
| 4 - 1 | Moment MY (Nm)    | -1211.2997 | 1211.2994  | 0.0          |

**CONCLUSION:**

Exact agreement of results (*taking into account different sign convention*).



## VERIFICATION EXAMPLE

### Plane truss with nodal loads - SSSL11

**Name of the test:**

SSLL11

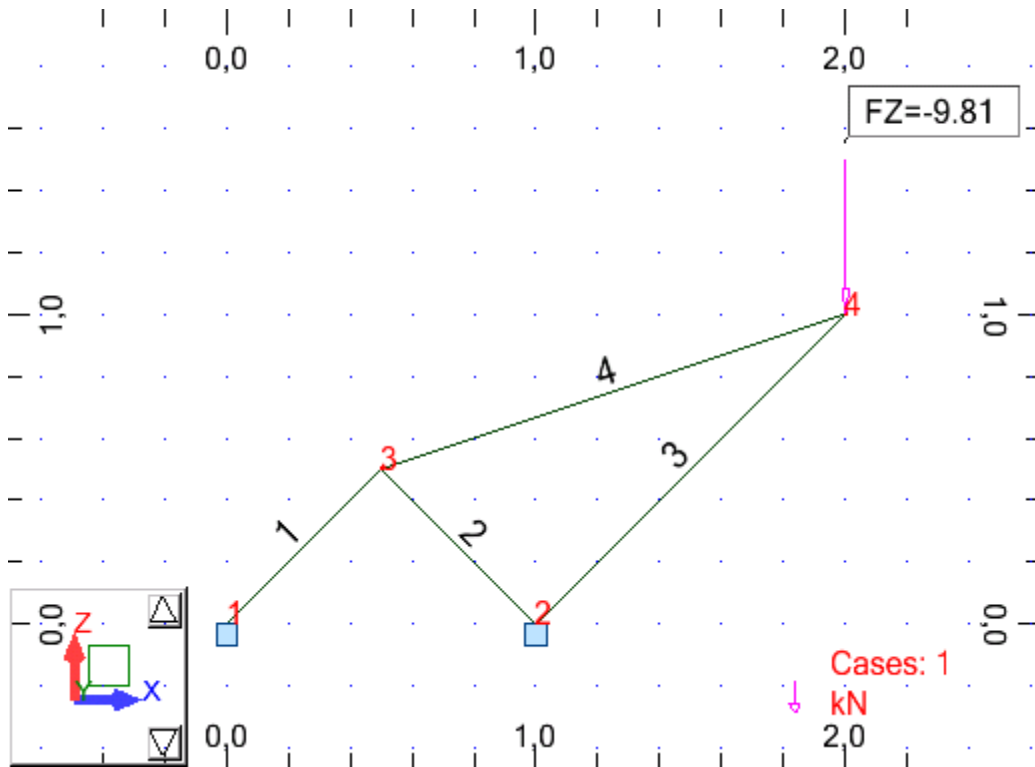
**Reference:**

AFNOR

**Specification:**

Truss made of four bars of different cross sections;  
material: elastic, linear, isotropic (non-compressible bars assumed).

**GEOMETRY:**



**DATA FILE:** SSSL11.rtd

**COMPARISON:**

| Node | Compared result     | RSA         | AFNOR       | Difference % |
|------|---------------------|-------------|-------------|--------------|
| 3    | Displacement UX (m) | 0.26517e-3  | 0.26517e-3  | 0.0          |
| 3    | Displacement UZ (m) | 0.08839e-3  | 0.08839e-3  | 0.0          |
| 4    | Displacement UX (m) | 3.47903e-3  | 3.47902e-3  | 0.0          |
| 4    | Displacement UZ (m) | -5.60035e-3 | -5.60084e-3 | 0.008        |

**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### Plane truss under thermal and displacement loadings - SSSL12

**Name of the test:**

SSLL12

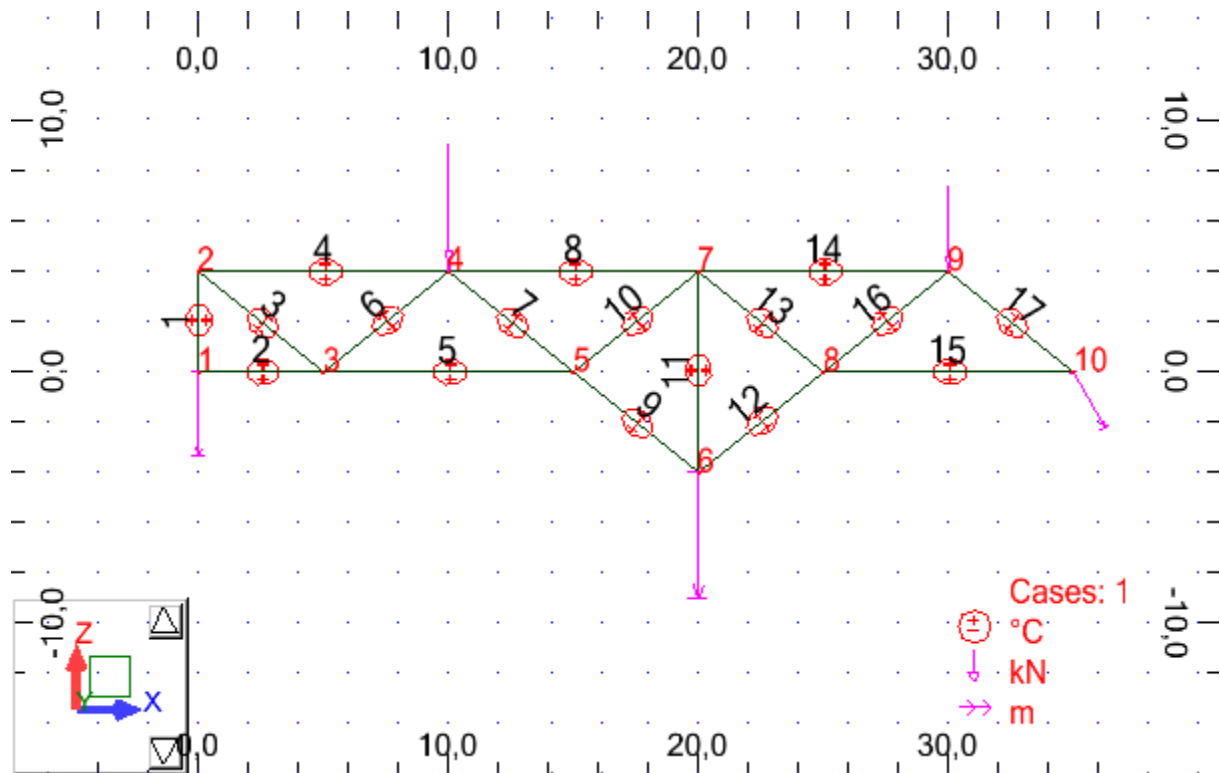
**Reference:**

AFNOR

**Specification:**

Plane truss - initial displacements - dilatation effect - pinned supports.

#### GEOMETRY:



**DATA FILE:**

SSLL12.rtd

#### COMPARISON:

| Node  | Compared result     | RSA      | AFNOR    | Difference % |
|-------|---------------------|----------|----------|--------------|
| 6 - 8 | Tension force (N)   | 43633    | 43633    | 0.0          |
| 8     | Displacement UZ (m) | -0.01618 | -0.01618 | 0.0          |

#### CONCLUSION:

Exact agreement of results.

## VERIFICATION EXAMPLE

### Shortening of a tie-beam - SSSL13

**Name of the test:**

SSLL13

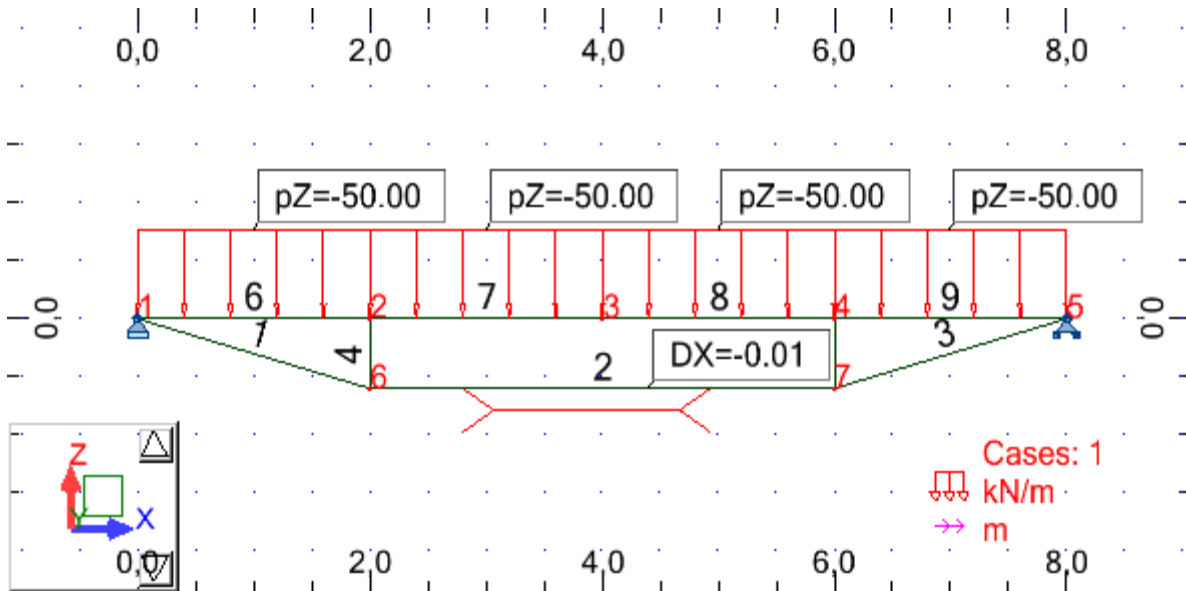
**Reference:**

AFNOR

**Specification:**

Simply supported beam with truss elements (hinged joints), shortening of bars; elastic, linear, isotropic material.

**GEOMETRY:**



**DATA FILE:**

SSLL13.rtd

**COMPARISON:**

| Node  | Compared result     | RSA         | AFNOR       | Difference % |
|-------|---------------------|-------------|-------------|--------------|
| 6 - 7 | Tension force (N)   | 584584      | 584584      | 0.0          |
| 3     | Moment MY (Nm)      | 49249.5     | 49249.5     | 0.0          |
| 2     | Displacement UZ (m) | -0.5428 e-3 | -0.5428 e-3 | 0.0          |

**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### Plane frame bending - SSSL14

**Name of the test:**

SSLL14

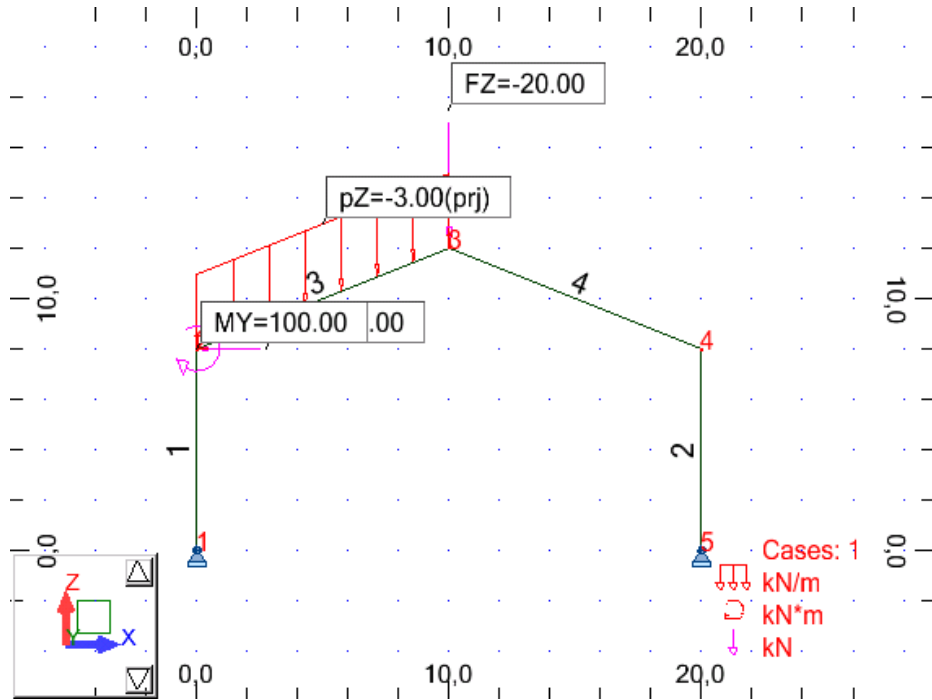
**Reference:**

AFNOR

**Specification:**

Simply supported symmetrical frame with asymmetric load; material: elastic, linear, isotropic (non-compressible bars assumed).

**GEOMETRY:**



**DATA FILE:**

SSLL14.rtd

**COMPARISON:**

| Node | Compared result         | RSA      | AFNOR    | Difference % |
|------|-------------------------|----------|----------|--------------|
| 1    | Vertical reaction (N)   | 31500.0  | 31500.0  | 0.0          |
| 1    | Horizontal reaction (N) | 20239.4  | 20239.4  | 0.0          |
| 3    | Displacement UZ (m)     | -0.03072 | -0.03072 | 0.0          |

**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### Beam on elastic (Winkler's) soil foundation - SSSL15

**Name of the test:**

SSSL15

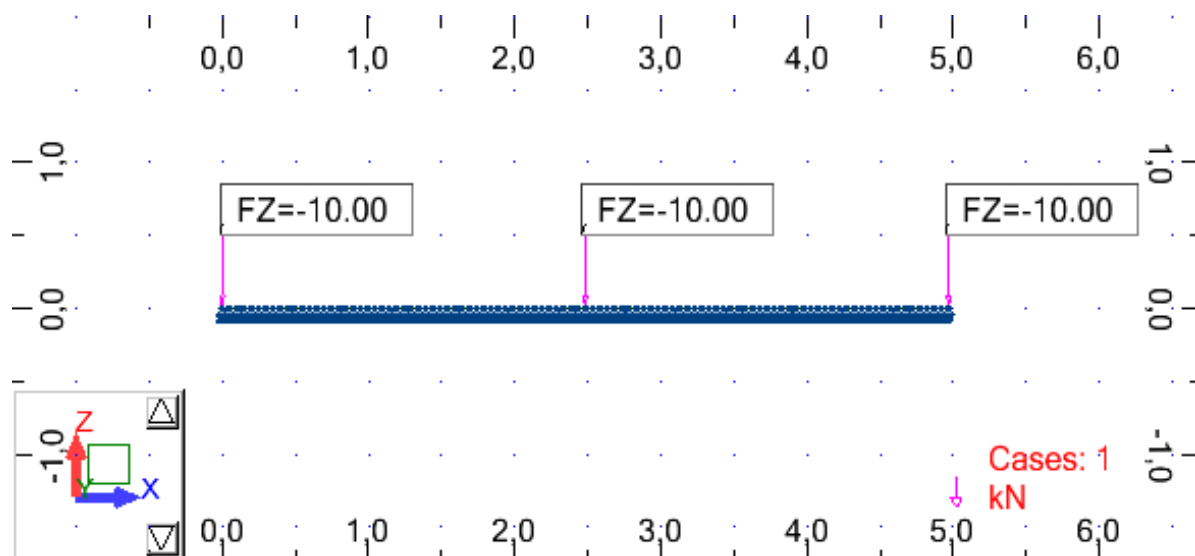
**Reference:**

AFNOR

**Specification:**

Simple beam on bidirectional, elastic foundation (Winkler's foundation)  
elastic, linear, isotropic material

**GEOMETRY:**



**DATA FILE:**

SSSL15.rtd (dense division on beam-elements with elastic supports),  
SSSL15R.rtd (2 Winkler's beam-elements, without nodal supports)

**COMPARISON:**

| Node | Compared result     | RSA<br>(SSSL15.rtd) | AFNOR     | Difference<br>% |
|------|---------------------|---------------------|-----------|-----------------|
| 51   | Moment MY (Nm)      | -5758               | 5759      | 0.017           |
| 51   | Displacement UZ (m) | -0.006843           | -0.006844 | 0.015           |
| 1    | Displacement UZ (m) | -0.007859           | -0.007854 | 0.064           |
| 1    | Rotation RY (rad)   | -0.000706           | -0.000706 | 0.0             |

| Node | Compared result     | RSA<br>(SSSL15R.rtd) | AFNOR     | Difference<br>% |
|------|---------------------|----------------------|-----------|-----------------|
| 2    | Moment MY (Nm)      | -5759                | 5759      | 0.0             |
| 2    | Displacement UZ (m) | -0.0068434           | -0.006844 | 0.009           |
| 1    | Displacement UZ (m) | -0.0078588           | -0.007854 | 0.061           |
| 1    | Rotation RY (rad)   | -0.000706            | -0.000706 | 0.0             |

**CONCLUSIONS:**

Excellent agreement of results.  
(taking into account different sign convention).

## VERIFICATION EXAMPLE

### Beam on elastic (Winkler's) soil foundation - SSSL16

**Name of the test:**

SSLL16

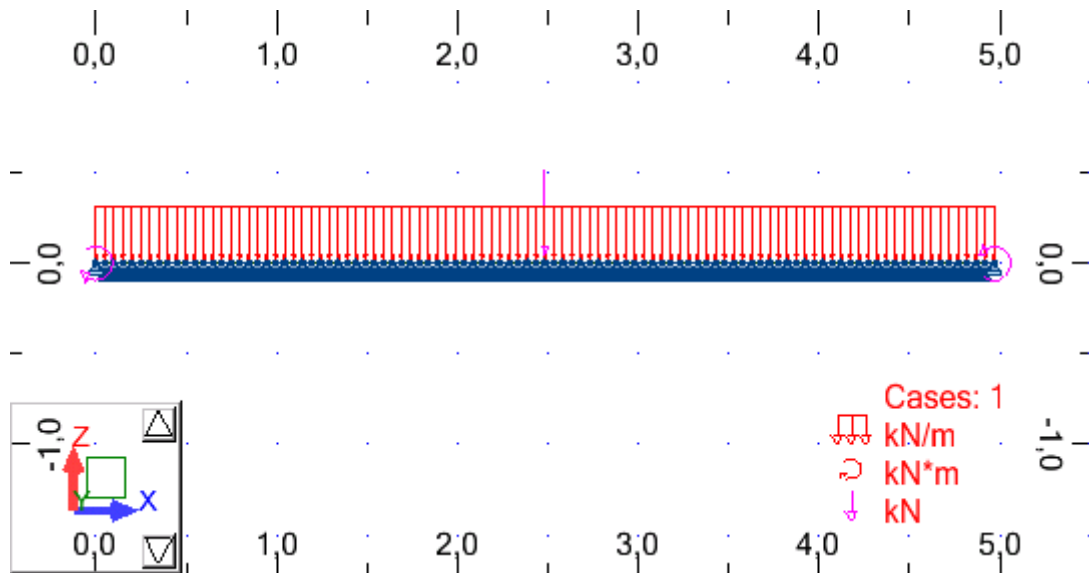
**Reference:**

AFNOR

**Specification:**

Simple beam on bidirectional, elastic foundation (Winkler's foundation)  
elastic, linear, isotropic material

**GEOMETRY:**



**DATA FILE:**

SSLL16.rtd (dense division on beam-elements with elastic supports),  
SSLL16R.rtd (2 Winkler's beam-elements, without nodal supports)

**COMPARISON:**

| Node | Compared result          | RSA<br>(SSLL16.rtd) | AFNOR       | Difference<br>% |
|------|--------------------------|---------------------|-------------|-----------------|
| 1    | Rotation RY (rad)        | -0.003045           | -0.003045   | 0.0             |
| 1    | Vertical reaction FZ (N) | 11675               | 11674       | 0.01            |
| 51   | Displacement UZ (m)      | -0.00423297         | -0.00423326 | 0.01            |
| 51   | Moment MY (Nm)           | -33839              | -33840      | 0.0             |

| Node | Compared result          | RSA<br>(SSLL16R.rtd) | AFNOR       | Difference<br>% |
|------|--------------------------|----------------------|-------------|-----------------|
| 1    | Rotation RY (rad)        | -0.003045            | -0.003045   | 0.0             |
| 1    | Vertical reaction FZ (N) | 11674                | 11674       | 0.0             |
| 2    | Displacement UZ (m)      | -0.00423299          | -0.00423326 | 0.01            |
| 2    | Moment MY (Nm)           | -33840               | -33840      | 0.0             |

**CONCLUSIONS:**

Excellent agreement of results.  
(taking into account different sign convention).



## 2. PLATE/SHELL STRUCTURES

## VERIFICATION EXAMPLE

### Rectangular membrane under in-plane shear - SSLP01

**Name of the test:**

SSLP01

**Reference:**

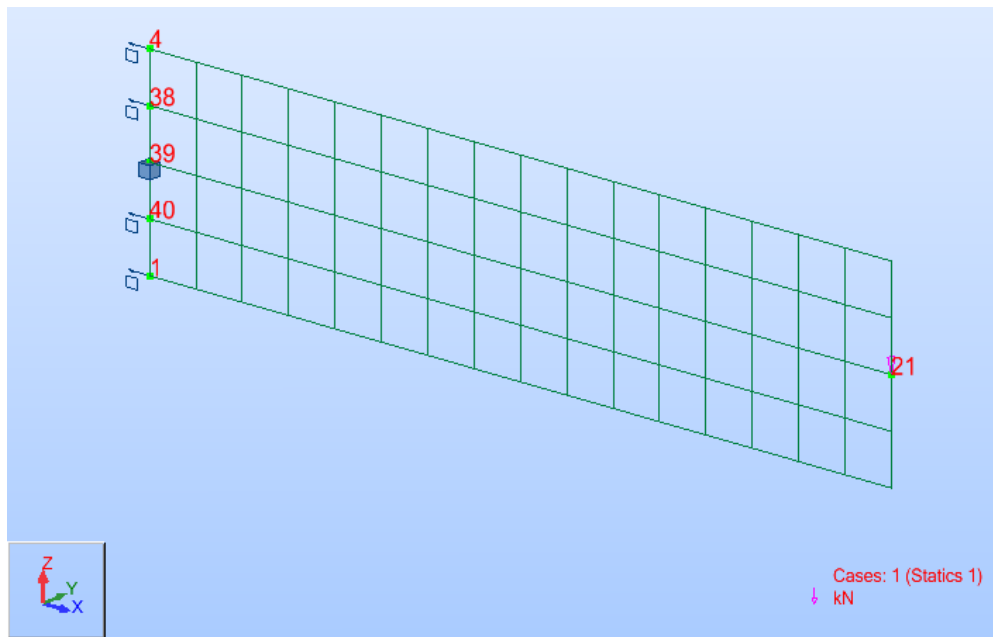
AFNOR

**Specification:**

Rectangular shell: in-plane bending and shear.

**GEOMETRY:**

Mesh 4x16 (3mm size square FE), point load in node 21



**DATA FILE**

SSLP01.rtd

**COMPARISON:**

| Node | Compared result             | RSA    | AFNOR  | Difference % |
|------|-----------------------------|--------|--------|--------------|
| 21   | Displacement UZ (mm)        | 0.3582 | 0.3573 | 0.252        |
| 4    | Stress (N/mm <sup>2</sup> ) | -79.56 | -80.0  | 0.550        |

**CONCLUSION:**

Very good agreement of results.

## VERIFICATION EXAMPLE

### Tension of perforated membrane - SSLP02

**Name of the test:**

SSLP02

**Reference:**

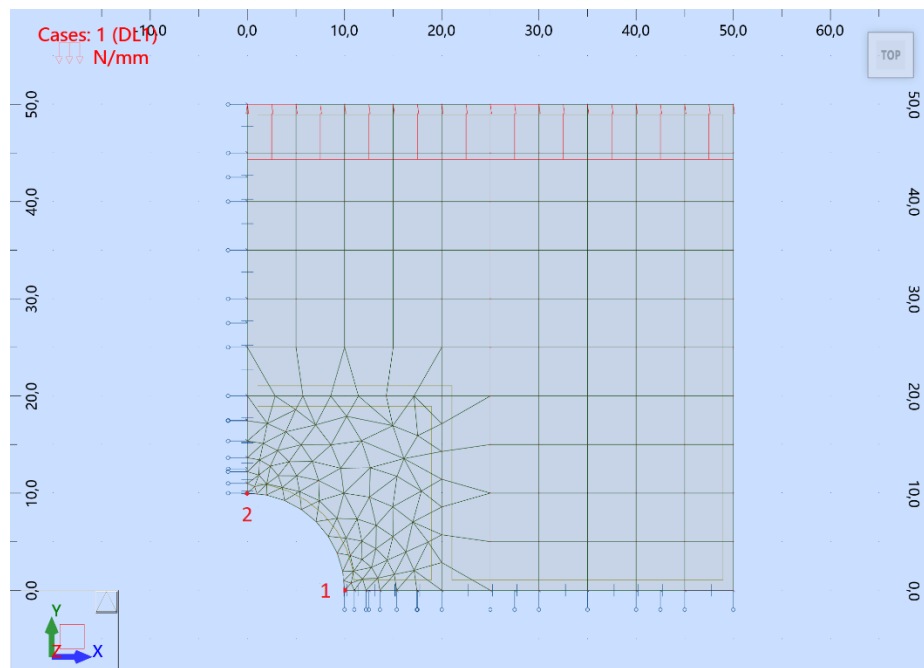
AFNOR

**Specification:**

Simple tension of perforated membrane.

**GEOMETRY:**

¼ of a model analyzed (due to symmetry) with a mesh 10x10



**DATA FILE**

SSLP02.rtd

**COMPARISON:**

| Node | Compared result                                     | RSA<br>Syy Polar (0,0,0) | AFNOR  | Difference<br>% |
|------|---|--------------------------|--------|-----------------|
| 1    | Stress $\sigma_{\theta\theta}$ (N/mm <sup>2</sup> ) | 7.40                     | 7.50   | 1.33            |
| 2    | Stress $\sigma_{\theta\theta}$ (N/mm <sup>2</sup> ) | - 2.49                   | - 2.50 | 0.04            |

**CONCLUSION:**

Good agreement of results.

## VERIFICATION EXAMPLE

### Rectangular plate: cantilever slab - SSLS01

**Name of the test:**

SSLS01

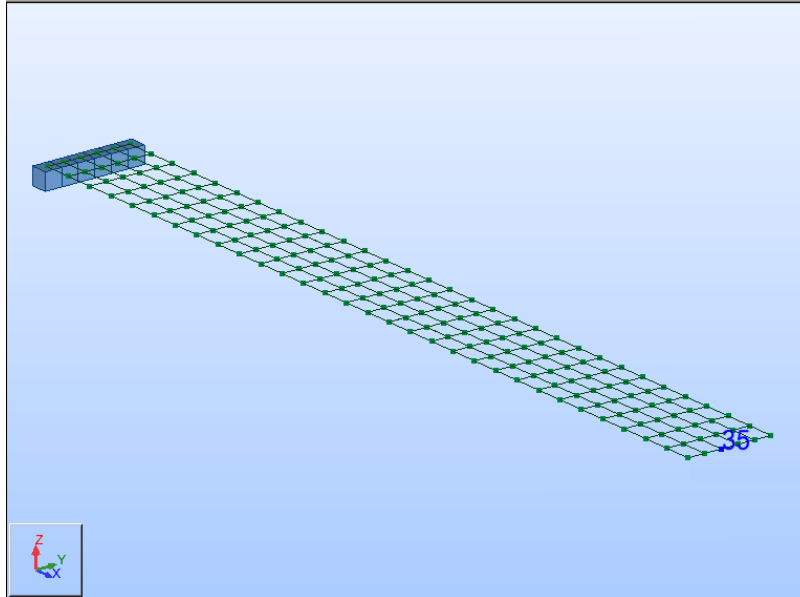
**Reference:**

AFNOR

**Specification:**

Cantilever slab under uniform pressure

#### GEOMETRY:



**DATA FILE**

SSLS01.rtd

#### COMPARISON:

| Node | Compared result      | RSA      | AFNOR   | Difference % |
|------|----------------------|----------|---------|--------------|
| 35   | Displacement UZ (mm) | - 95.919 | - 95.90 | 0.021        |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Simply supported square plate - SSLS02

**Name of the test:**

SSLS02

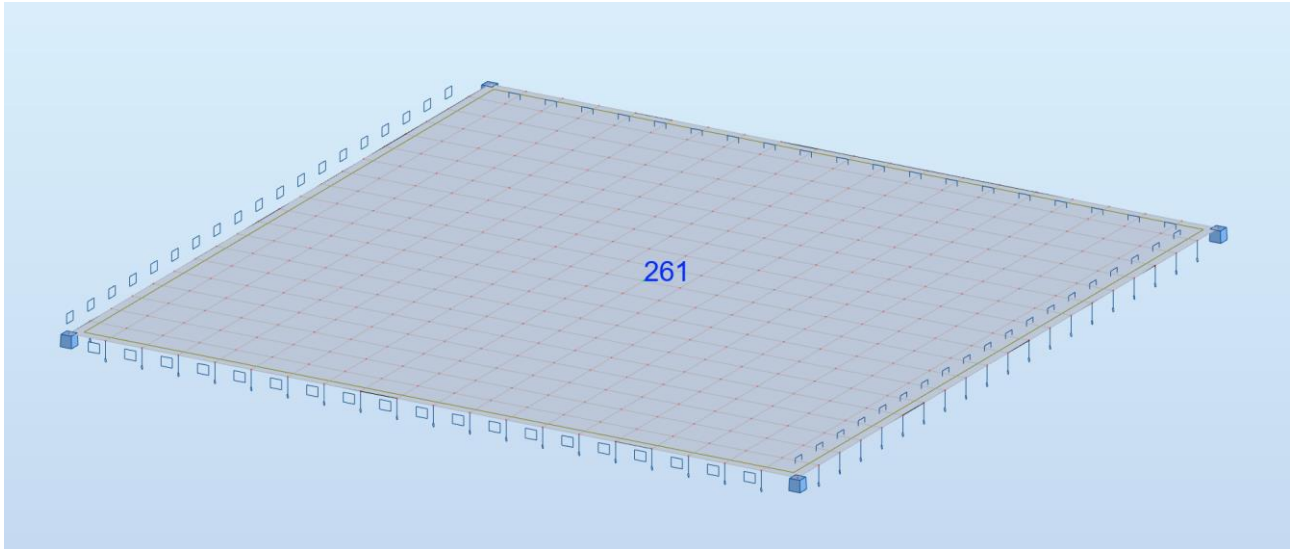
**Reference:**

AFNOR

**Specification:**

Simply supported square plate under self weight.

#### GEOMETRY:



**DATA FILE:**

SSLS02.rtd

#### COMPARISON:

| Node | Compared result      | RSA     | AFNOR     | Difference % |
|------|----------------------|---------|-----------|--------------|
| 261  | Displacement UZ (mm) | -0,1647 | - 0.1648* | 0.06         |

\* "Guide..." presents an incorrect value (compare with SSLS 24)

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Circular plate under uniform load - SSLS03

**Name of the test:**

SSLS03

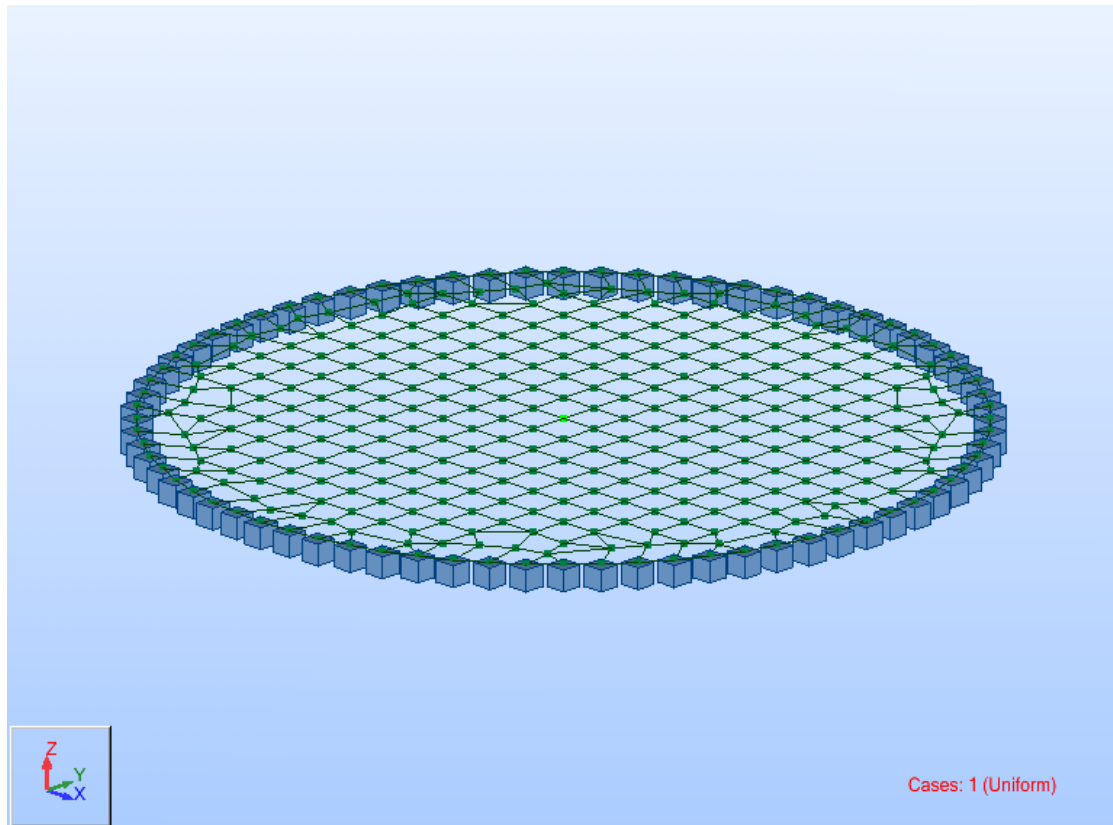
**Reference:**

AFNOR

**Specification:**

Circular plate with clamped edges under uniform load

**GEOMETRY:**



**DATA FILES:** SSLS03.rtd

**COMPARISON:**

| Node | Compared result      | RSA     | AFNOR   | Difference % |
|------|----------------------|---------|---------|--------------|
| 1    | Displacement UZ (mm) | - 6.477 | - 6.500 | 0.36         |

**CONCLUSION:**

Very good agreement of results.

## VERIFICATION EXAMPLE

### Beam of Z-section (using shell elements) - SSLS04

**Name of the test:**

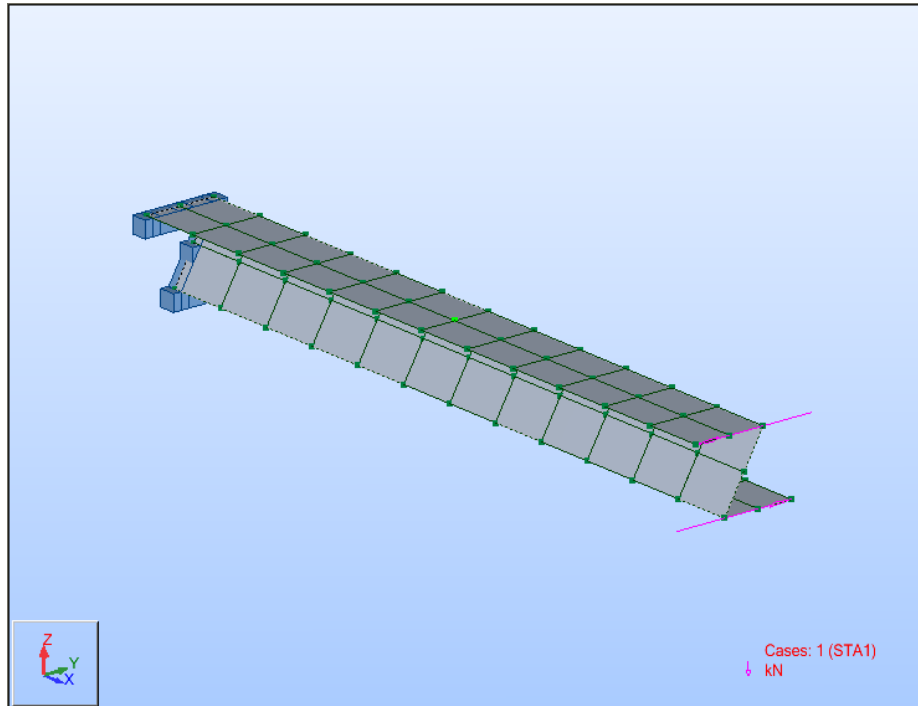
SSLS04

**Reference:**

AFNOR

**GEOMETRY:**

---



**DATA FILE:** SSLS04.rtd

---

**COMPARISON:**

---

| Node | Compared result      | RSA    | AFNOR   | Difference % |
|------|----------------------|--------|---------|--------------|
| 117  | Displacement UY (mm) | - 8.93 | - 7.150 | 24.895       |

**CONCLUSION:**

---

Poor agreement of results.

## VERIFICATION EXAMPLE

### Box section in torsion (using shell elements) - SSSL05

**Name of the test:**

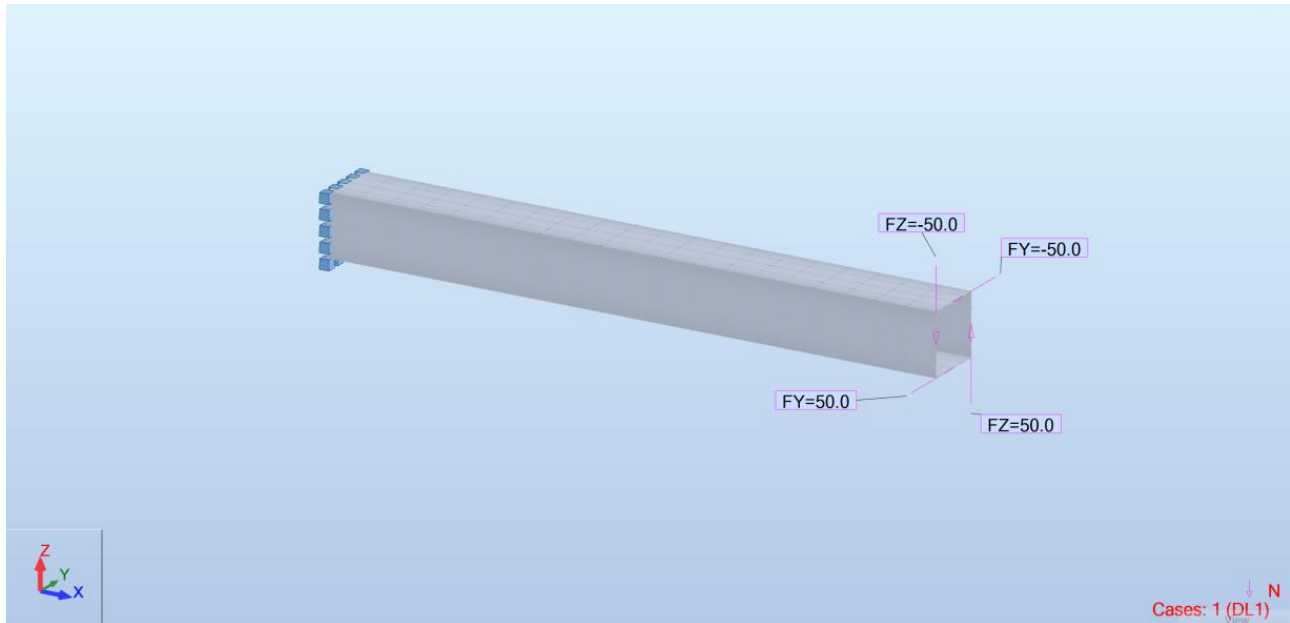
SSLS05

**Reference:**

AFNOR

**Specification:** Shell - Box section - Shear - Torsion.

#### GEOMETRY:



**DATA FILE:** SSSL05.rtd

#### COMPARISON:

| Node | Compared result     | RSA         | AFNOR       | Difference % |
|------|---------------------|-------------|-------------|--------------|
| 158  | Displacement UY (m) | - 0.616 e-6 | - 0.617 e-6 | 0.17         |
| 158  | Rotation RX (rad)   | 0.1232 e-4  | 0.123 e-4   | 0.15         |
| 83   | Displacement UZ (m) | - 0.986 e-6 | - 0.987 e-6 | 0.09         |
| 83   | Rotation RX (rad)   | 0.1972 e-4  | 0.197 e-4   | 0.11         |

#### CONCLUSION:

Excellent agreement of results.



## VERIFICATION EXAMPLE

### Thin-walled cylinder under uniform radial pressure - SSSL06

**Name of the test:**

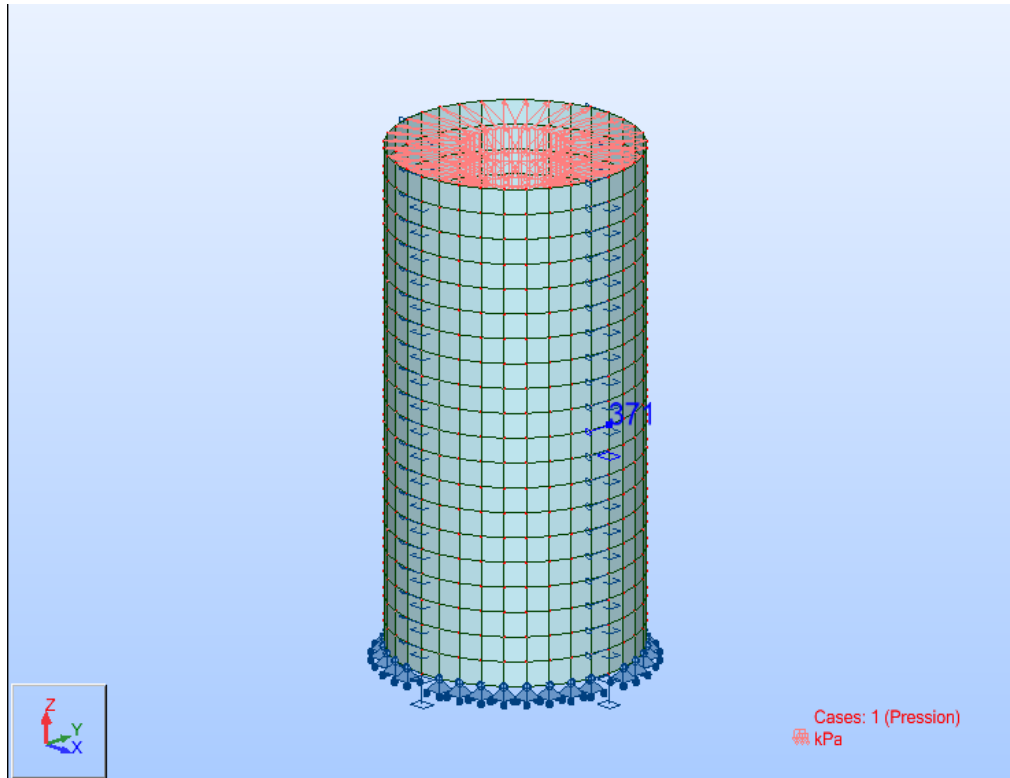
SSLS06

**Reference:**

AFNOR

**Specification:** Shell - Cylinder - Material: elastic - Pressure

#### GEOMETRY:



**DATA FILE:** SSSL06.rtd

#### COMPARISON:

| Node | Compared result         | RSA         | AFNOR       | Difference % |
|------|-------------------------|-------------|-------------|--------------|
| 371  | Displacement UX (mm)    | 2.371 e-3   | 2.380 e-3   | 0.378        |
| 371  | Circumfer. stress [kPa] | 498.1       | 500.0       | 0.380        |
| 741  | Displacement UZ (mm)    | - 2.964 e-3 | - 2.860 e-3 | 1.036        |

#### CONCLUSION:

Good agreement of results.

## VERIFICATION EXAMPLE

### Thin-walled cylinder with uniform axial load - SSLS07

**Name of the test:**

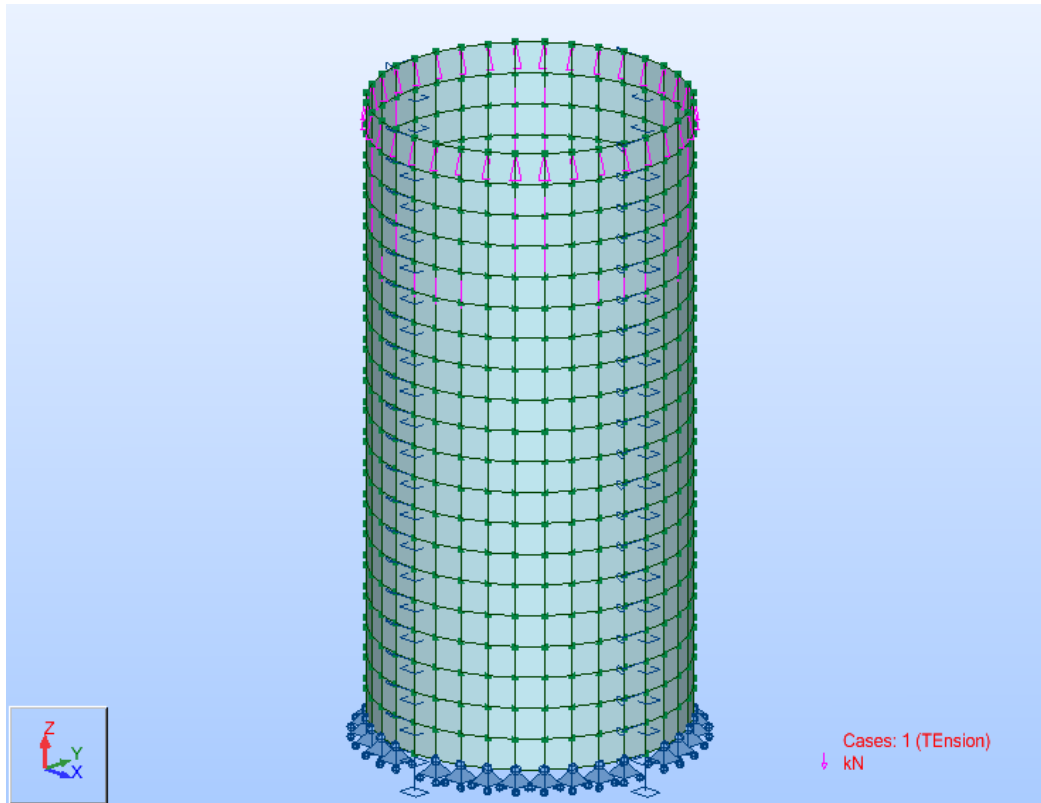
SSLS07

**Reference:**

AFNOR

**Specification:** Shell - Material: elastic - uniform load - Cylinder

#### GEOMETRY:



**DATA FILE:** SSLS07.rtd

#### COMPARISON:

| Node | Compared result      | RSA         | AFNOR      | Difference % |
|------|----------------------|-------------|------------|--------------|
| 371  | Displacement UX (mm) | - 7.152 e-4 | -7.140 e-4 | 0.168        |
| 371  | Tension stress (kPa) | 500.6       | 500.0      | 0.120        |
| 741  | Displacement UZ (mm) | 9.626 e-3   | 9.520 e-3  | 1.113        |

#### CONCLUSION:

Very good agreement of results.

## VERIFICATION EXAMPLE

### Thin-walled cylinder under hydrostatic pressure - SSLS08

**Name of the test:**

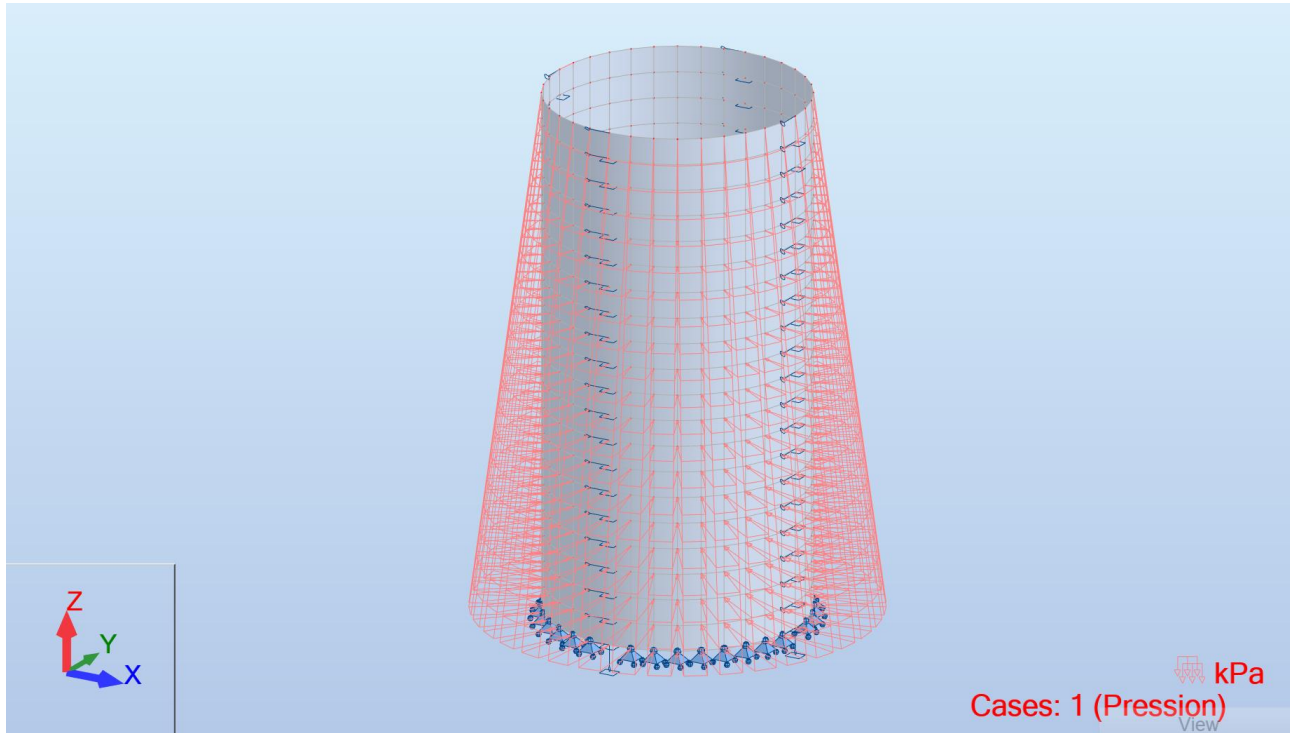
SSLS08

**Reference:**

AFNOR

**Specification:** Shell - Material: elastic - Hydrostatic pressure - Cylinder

#### GEOMETRY:



**DATA FILE:** SSLS08.rtd

#### COMPARISON:

| Node | Compared result      | RSA        | AFNOR      | Difference % |
|------|----------------------|------------|------------|--------------|
| 371  | Displacement UX (mm) | 2.371 e-3  | 2.380 e-3  | 0.379        |
| 371  | Radial stress (kPa)  | 498.1      | 500.0      | 0.380        |
| 741  | Displacement UZ (mm) | -2.964 e-3 | -2.860 e-3 | 0.489        |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Thin-walled cylinder under self-weight - SSLS09

**Name of the test:**

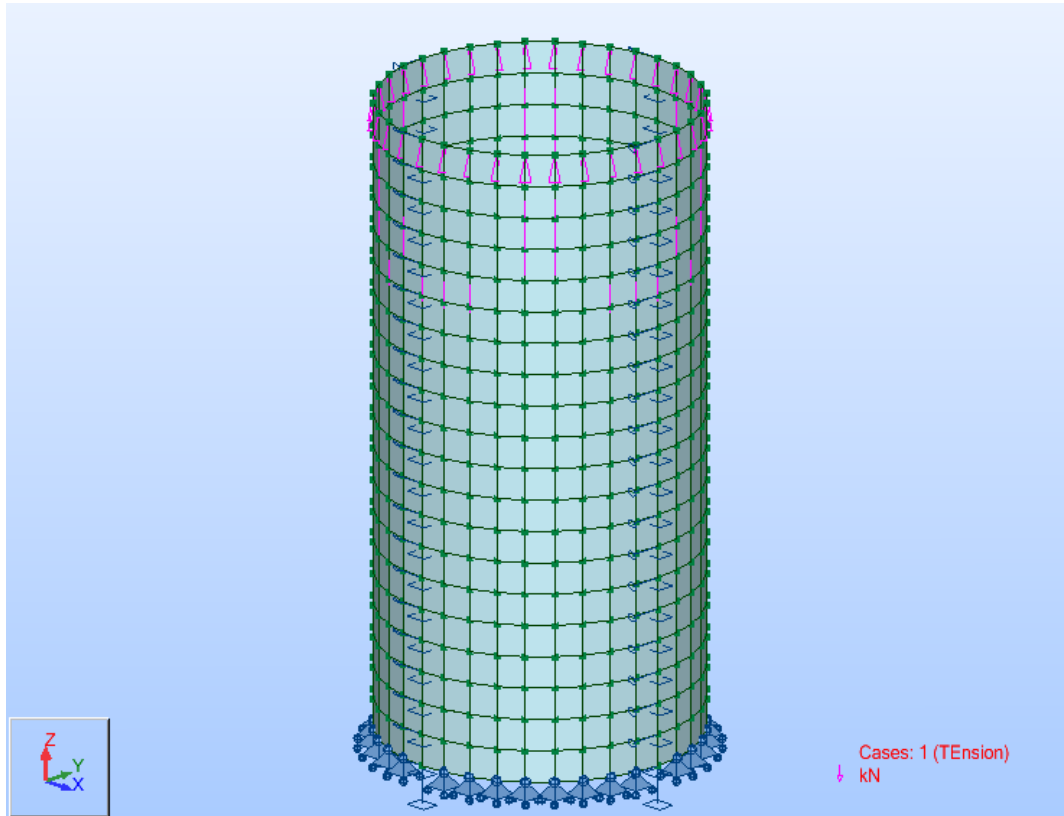
SSLS09

**Reference:**

AFNOR

**Specification:** Shell - Material: elastic – Self-weight - Cylinder

#### GEOMETRY:



**DATA FILE:** SSLS09.rtd

#### COMPARISON:

| Node | Compared result         | RSA        | AFNOR      | Difference % |
|------|-------------------------|------------|------------|--------------|
| 371  | Displacement UX (mm)    | 0.2243 e-3 | 0.2245 e-3 | 0.089        |
| 38   | Circumfer. stress (kPa) | 312.5      | 314.2      | 0.637        |
| 741  | Displacement UZ (mm)    | -3.019 e-3 | -3 e-3     | 0.956        |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Torus under uniform internal pressure - SSLS10

**Name of the test:**

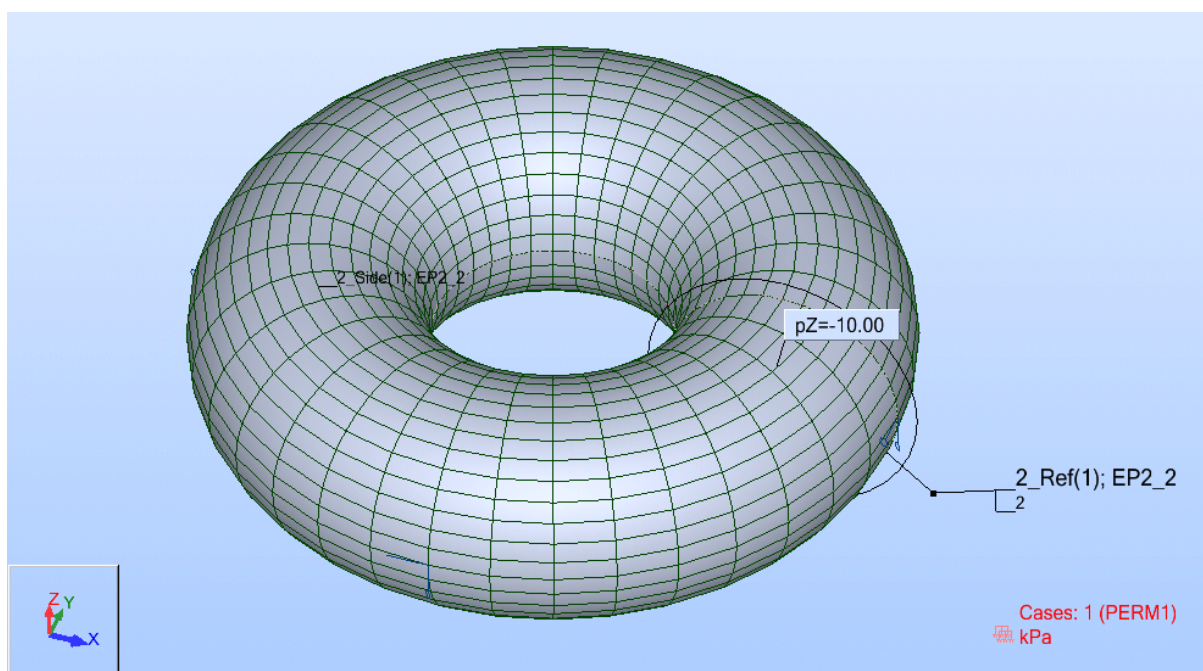
SSLS10

**Reference:**

AFNOR

**Codification:** Shell - Torus - Material: elastic - Pressure.

#### GEOMETRY:



**DATA FILE:** SSLS10.rtd

#### COMPARISON:

| Node              | Compared result                      | RSA       | AFNOR    | Difference % |
|-------------------|--------------------------------------|-----------|----------|--------------|
| 335<br>(internal) | Displacement UY (m)                  | 0.7906e-7 | 1.19 e-7 | 33,56        |
|                   | Horizontal stress $\sigma_{xx}$ (Pa) | 2.389e+5  | 2.50 e+5 | 4,44         |
|                   | Vertical stress $\sigma_{yy}$ (Pa)   | 7.355e+5  | 7.50 e+5 | 1,93         |
| 362<br>(external) | Displacement UY (m)                  | 1.934e-6  | 1.79 e-6 | 8,04         |
|                   | Horizontal stress $\sigma_{xx}$ (Pa) | 2.601e+5  | 2.50 e+5 | 4,04         |
|                   | Vertical stress $\sigma_{yy}$ (Pa)   | 4.156e+5  | 4.17 e+5 | 0,34         |

#### CONCLUSION:

Good agreement of results.

## VERIFICATION EXAMPLE

### Thin-walled cone subjected to uniform internal pressure - SSSL11

**Name of the test:**

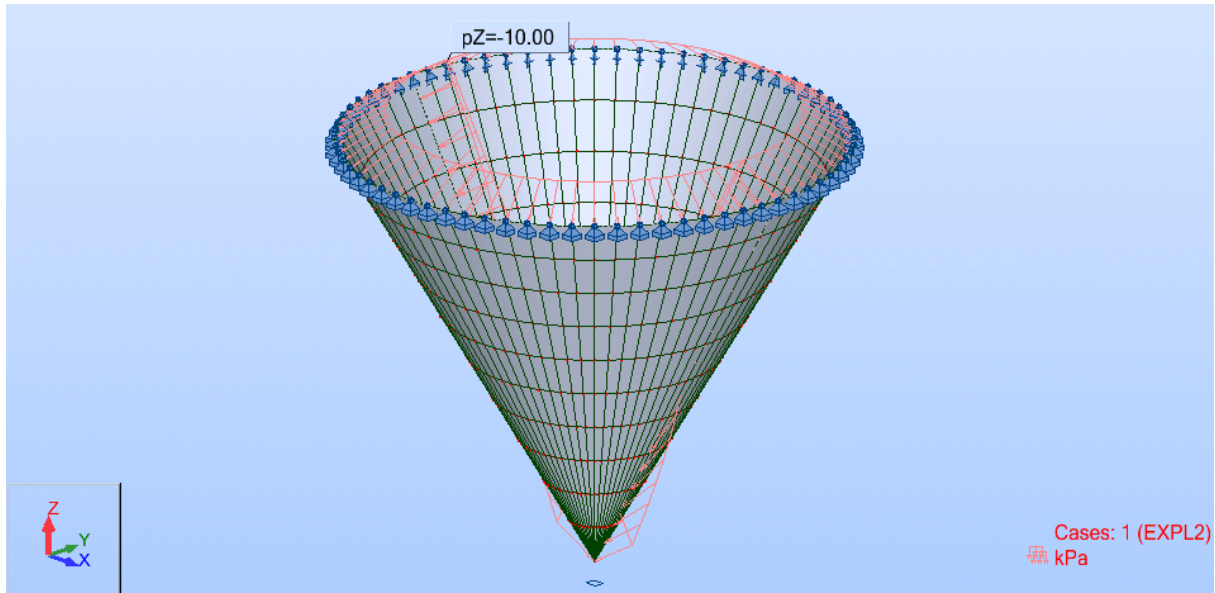
SSLS11

**Reference:**

AFNOR

**Specification:** Shell - Cone - Material: elastic - Pressure.

#### GEOMETRY:



**DATA FILE:** SSSL11.rtd

#### COMPARISON:

| Node              | Compared result                    | RSA        | AFNOR      | Difference % |
|-------------------|------------------------------------|------------|------------|--------------|
| 7<br>(mid-height) | Vertical stress (Pa)               | 1.45 e+5   | 1.44 e+5   | 0.69         |
|                   | Horizontal stress (Pa)             | 2.88 e+5   | 2.89 e+5   | 0.03         |
|                   | Displacement UX ( $\delta_R$ ) (m) | 0.5843 e-6 | 0.5842 e-6 | 0.02         |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Spherical shell subjected to a pressure - SSSL14

**Name of the test:**

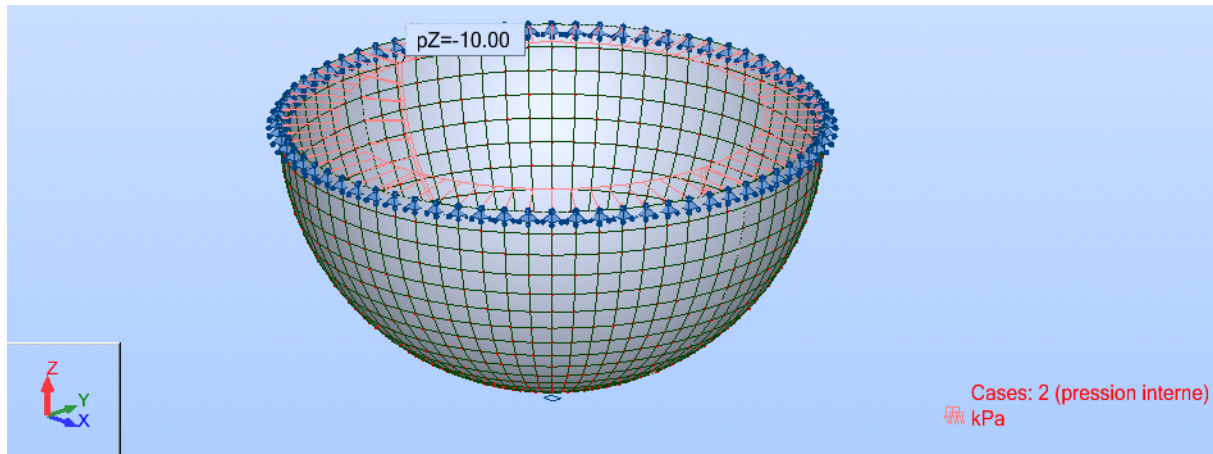
SSLS14

**Reference:**

AFNOR

**Codification:** Shell - spherical cup - Material: elastic - Uniform pressure

#### GEOMETRY:



**DATA FILE:** SSSL14.rtd

#### COMPARISON:

| Node | Compared result             | RSA       | AFNOR    | Difference % |
|------|-----------------------------|-----------|----------|--------------|
| All  | Horizontal stress (Pa)      | 2.499 e+5 | 2.50 e+5 | 0.12         |
| 1    | Displacement $\delta_R$ (m) | 8.33 e-7  | 8.33 e-7 | 0.0          |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION PROBLEM

### Spherical shell subjected to a moment - SSLS16

**Name of the test:**

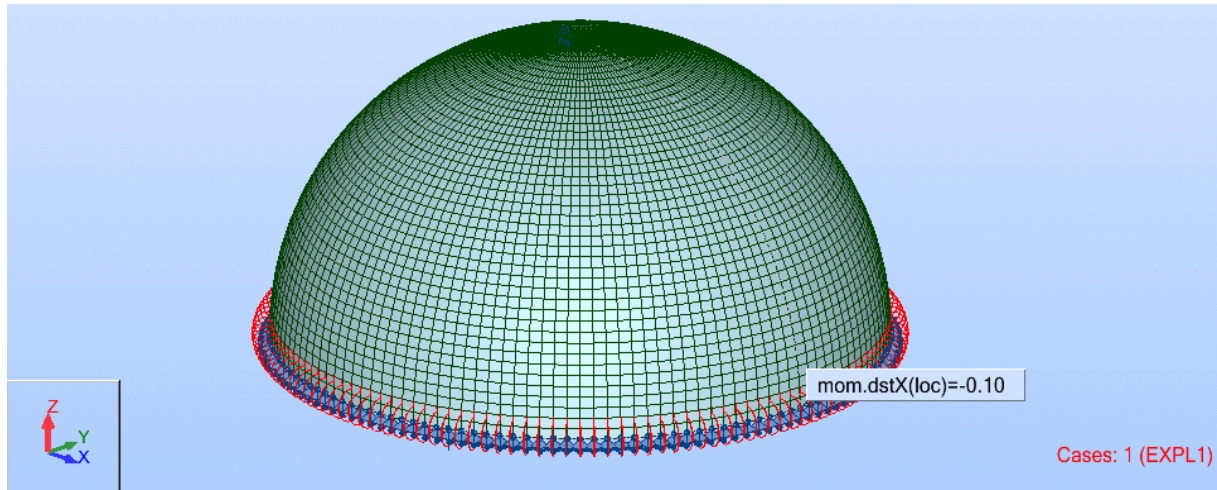
SSLS16

**Reference:**

AFNOR

**Codification:** Shell - spherical cup - Material: elastic - Uniform moment

#### GEOMETRY:



**DATA FILE:** SSLS16.rtd

#### COMPARISON:

| Node | Compared result             | RSA      | AFNOR    | Difference % |
|------|-----------------------------|----------|----------|--------------|
| 692  | Horizontal stress (Pa)      | 8.34 e+5 | 8.26 e+5 | 0.96         |
|      | Displacement $\delta_R$ (m) | 3.93 e-6 | 3.93 e-6 | 0.0          |

#### CONCLUSION:

Excellent agreement of results.



## VERIFICATION EXAMPLE

### Spherical shell - SSLS17

**Name of the test:**

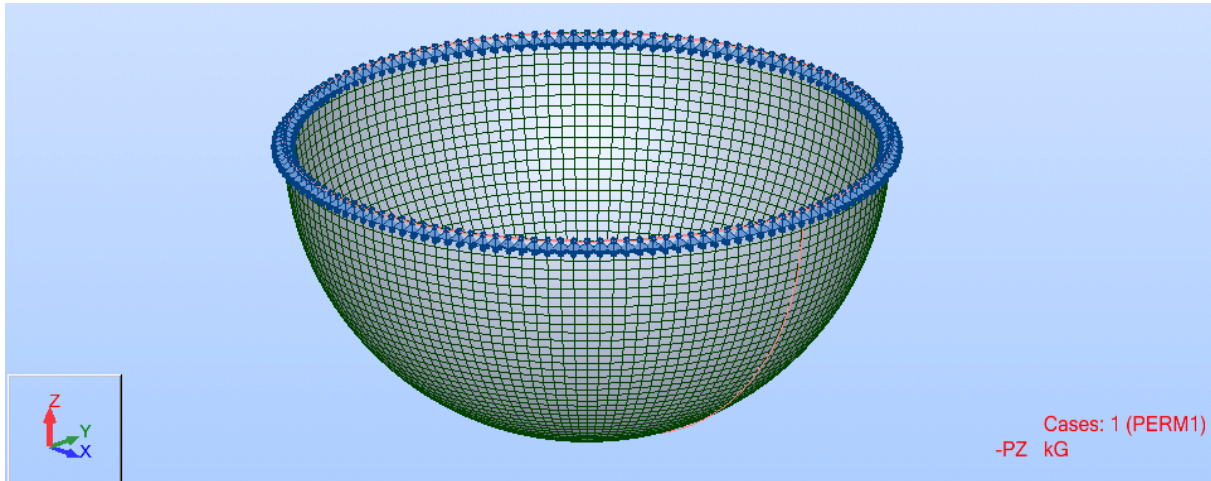
SSLS17

**Reference:**

AFNOR

**Codification:** Shell - spherical cup - Material: elastic – Self weight

#### GEOMETRY:



**DATA FILE:** SSLS17.rtd

#### COMPARISON:

| Node | Compared result             | RSA        | AFNOR      | Difference % |
|------|-----------------------------|------------|------------|--------------|
| 1    | Horizontal stress (Pa)      | - 8.19 e+4 | 7.85 e+4   | 4.33         |
|      | Vertical stress (Pa)        | 7.59 e+4   | - 7.85 e+4 | 3.31         |
|      | Displacement $\delta_R$ (m) | 4.99 e-7   | 4.86 e-7   | 2.67         |

#### CONCLUSION:

Good agreement of results.

## VERIFICATION EXAMPLE

### Cylindrical shell subjected to concentrated force - SSLS20

**Name of the test:**

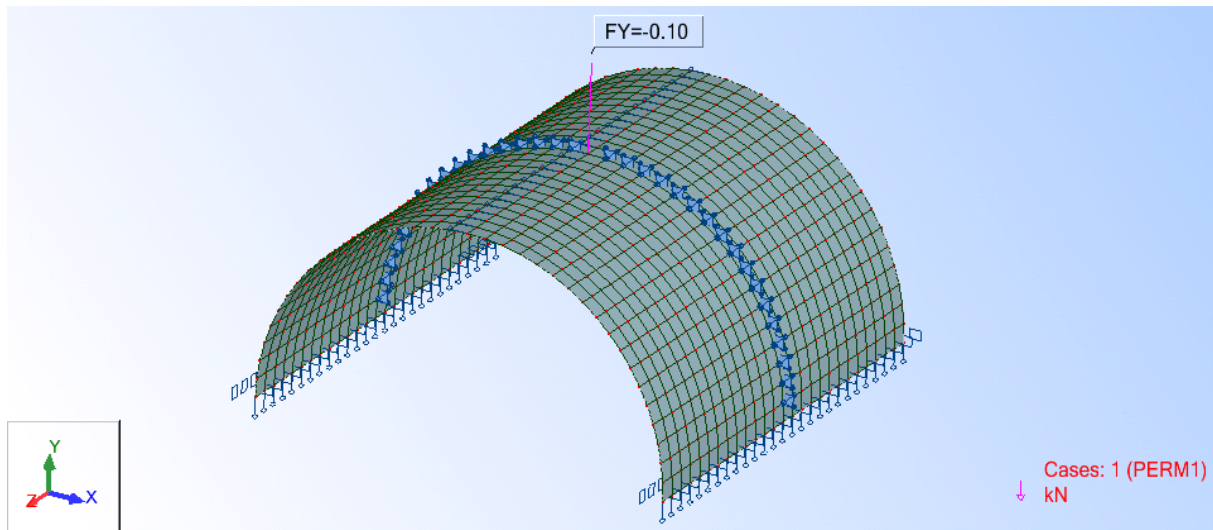
SSLS20

**Reference:**

AFNOR

**Codification:** Cylindrical shell - Material: elastic - Concentrated forces.

#### GEOMETRY:



**DATA FILE:** SSLS20.rtd

#### COMPARISON:

| Node | Compared result     | RSA         | AFNOR       | Difference % |
|------|---------------------|-------------|-------------|--------------|
| 500  | Displacement UY (m) | -11.374 e-2 | -11.390 e-2 | 0.14         |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Spherical shell with an opening - SSLS21

**Name of the test:**

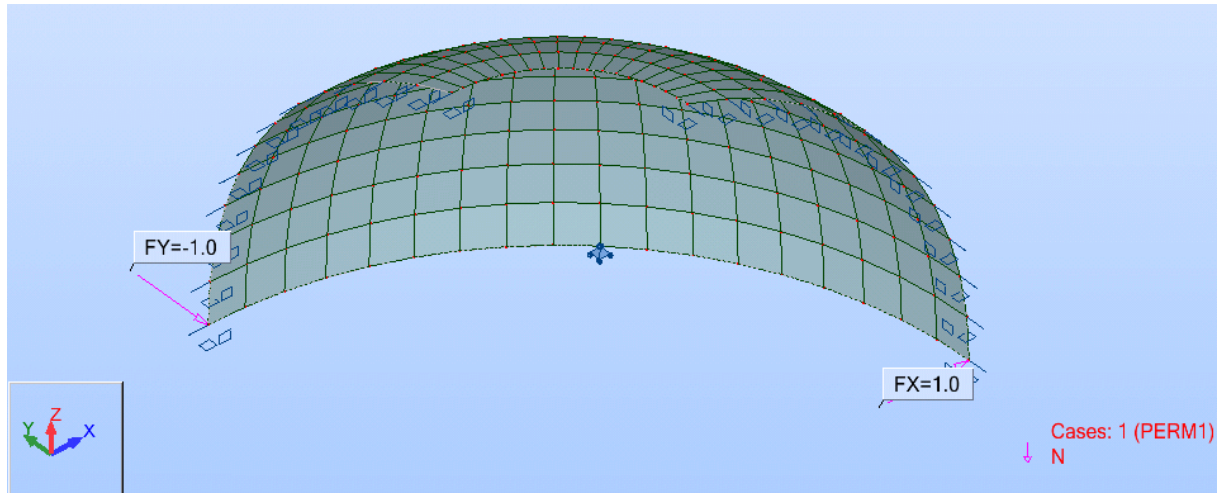
SSLS21

**Reference:**

AFNOR

**Codification:** Spherical shell - Material: elastic - Concentrated forces.

#### GEOMETRY:



**DATA FILE:** SSLS21.rtd

#### COMPARISON:

| Node | Compared result     | RSA        | AFNOR     | Difference % |
|------|---------------------|------------|-----------|--------------|
| 1    | Displacement UX (m) | 101.47 e-3 | 94.00 e-3 | 7.95         |

#### CONCLUSION:

Results correct.

## VERIFICATION EXAMPLE

### Spherical dome subjected to uniform external pressure - SSLS22

**Name of the test:**

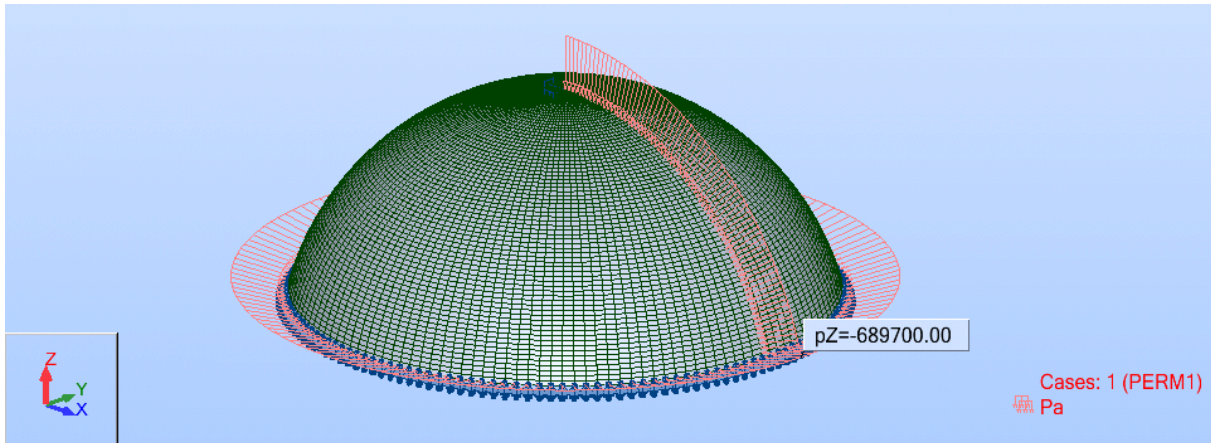
SSLS22

**Reference:**

AFNOR

**Specification:** Spherical shell - Material: elastic - pressure.

#### GEOMETRY:



**DATA FILE:** SSLS22.rtd

#### COMPARISON:

| Node | Compared result                    | RSA       | AFNOR     | Difference % |
|------|------------------------------------|-----------|-----------|--------------|
| 5794 | Displacement UX (m)                | 1.74 e-3  | 1.73 e-3  | 0.58         |
| 5824 | Displacement UX (m)                | 1.02 e-3  | 1.02 e-3  | 0.0          |
| 5794 | Vertical stress $\sigma_{YY}$ (Pa) | -0.68 e+8 | -0.74 e+8 | 8.11         |
| 5824 | Vertical stress $\sigma_{YY}$ (Pa) | -0.69 e+8 | -0.68 e+8 | 1.47         |

#### CONCLUSION:

Results correct.

## VERIFICATION EXAMPLE

### Cylindrical membrane subjected to bending - SSLS23

**Name of the test:**

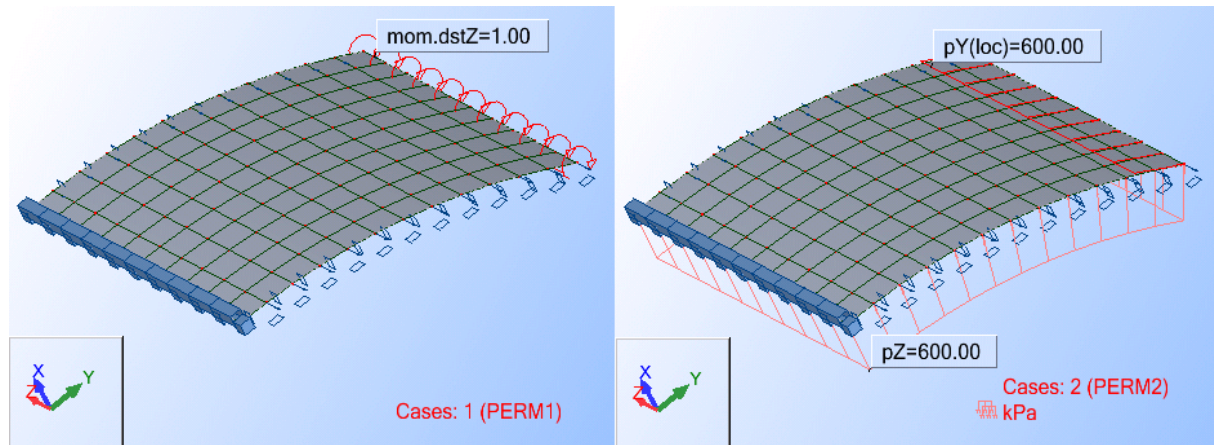
SSLS23

**Reference:**

AFNOR

**Specification:** Bending - Membrane effect

#### GEOMETRY:



**DATA FILE:** SSLS23.rtd

#### COMPARISON:

| Case         | Node | Compared result                                     | RSA   | AFNOR | Difference % |
|--------------|------|---|-------|-------|--------------|
| 1 (flexion)  | 87   | Stress $\sigma_{xx}$ (MPa)<br>Dir Y, External layer | 60.00 | 60.00 | 0.00         |
| 2 (membrane) | 87   | Stress $\sigma_{xx}$ (MPa)<br>Direction Y           | 59.99 | 60.00 | 0.02         |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Simply supported rectangular plate with uniform load - SSLS24

Name of the test:

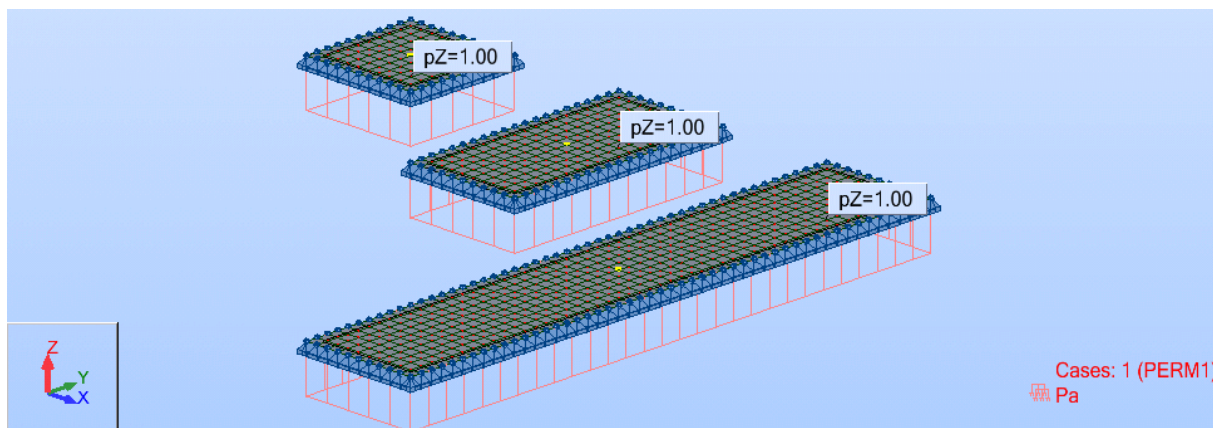
SSLS24

Reference:

AFNOR

Specification: Plate - Pressure - Simple support.

#### GEOMETRY:



DATA FILE: SSLS24.rtd

#### COMPARISON:

Case where  $b/a=1$ :

| Node | Compared result   | RSA   | AFNOR | Difference % |
|------|-------------------|-------|-------|--------------|
| 81   | Deflection (m)    | 44.35 | 44.30 | 0.10         |
| 81   | Moment Mxx (Nm/m) | 4.82  | 4.79  | 0.63         |
| 81   | Moment Myy (Nm/m) | 4.82  | 4.79  | 0.63         |

Case where  $b/a=2$ :

| Node | Compared result   | RSA    | AFNOR  | Difference % |
|------|-------------------|--------|--------|--------------|
| 267  | Deflection (m)    | 110.16 | 110.06 | 0.40         |
| 267  | Moment Mxx (Nm/m) | 10.20  | 10.17  | 0.32         |
| 267  | Moment Myy (Nm/m) | 4.63   | 4.64   | 0.19         |

Case where  $b/a=5$ :

| Node | Compared result   | RSA    | AFNOR  | Difference % |
|------|-------------------|--------|--------|--------------|
| 693  | Deflection (m)    | 140.53 | 141.60 | 0.75         |
| 693  | Moment Mxx (Nm/m) | 12.46  | 12.46  | 0.03         |
| 693  | Moment Myy (Nm/m) | 3.77   | 3.75   | 0.63         |

#### CONCLUSION:

Excellent agreement of results.



## VERIFICATION EXAMPLE

### Simply supported rectangular plate with bending moment - SSLS26

**Name of the test:**

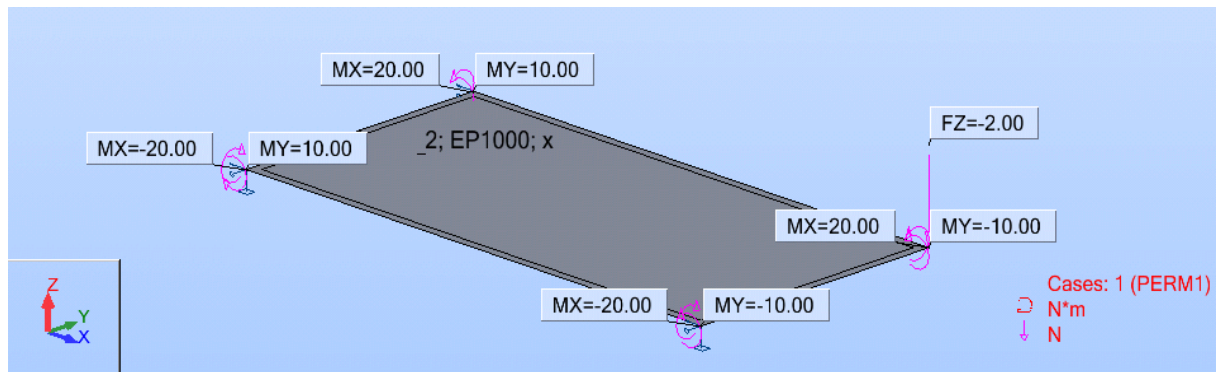
SSLS26

**Reference:**

AFNOR

**Specification:** Plate - Pressure - Simple support – Nodal moment

#### GEOMETRY:



**DATA FILE:** SSLS26.rtd

#### COMPARISON:

| Node | Compared result     | RSA    | AFNOR  | Difference % |
|------|---------------------|--------|--------|--------------|
| 1    | Displacement UZ (m) | -12.44 | -12.48 | 0.32         |

#### CONCLUSION:

Excellent agreement of results.



## VERIFICATION EXAMPLE

### Plate under perpendicular shear - SSLS27

**Name of the test:**

SSLS27

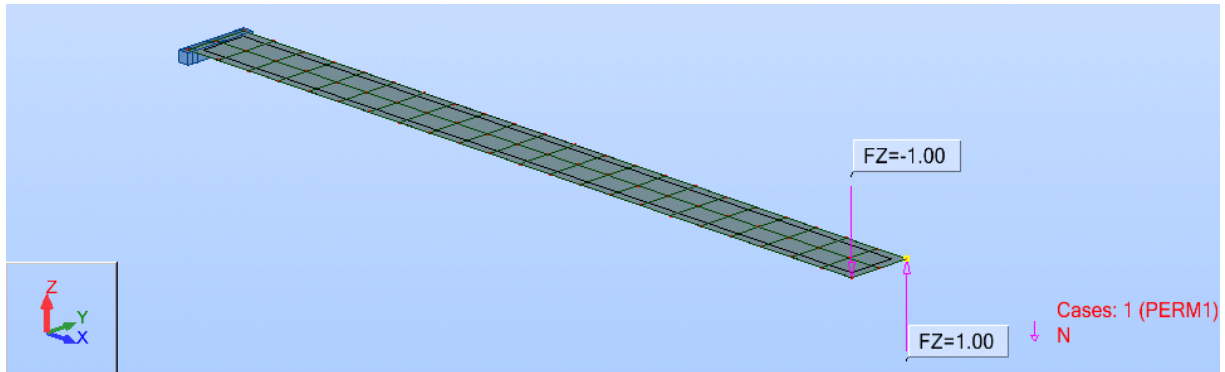
**Reference:**

AFNOR

**Specification:**

**Plate under perpendicular shear with one edge fixed**

#### GEOMETRY:



**DATA FILE:**

SSLS27.rtd

#### COMPARISON:

| Node | Compared result    | RSA       | AFNOR    | Difference % |
|------|--------------------|-----------|----------|--------------|
| 3    | Displacement Z (m) | 35.39 e-3 | 35.37e-3 | 0.06         |

#### CONCLUSION:

Excellent agreement of results.

## 3. VOLUMIC STRUCTURES

## VERIFICATION EXAMPLE

### Solid cylinder subjected to simple tension - SSLV01

**Name of the test:**

SSLV 01

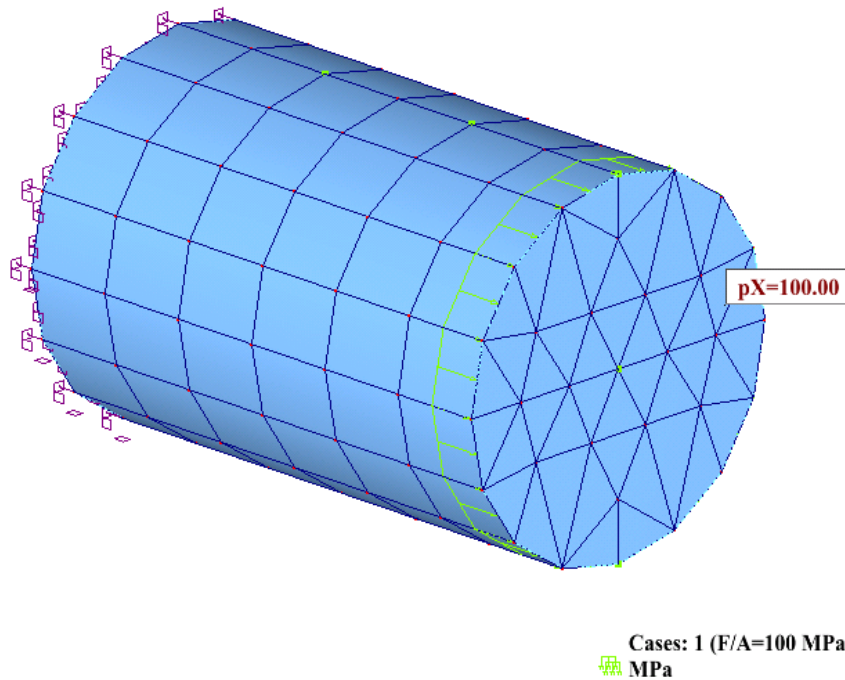
**Reference:**

AFNOR

**Specification:**

Solid cylinder - Tension - compression - Poisson's coefficient.

**GEOMETRY:**



**DATA FILE:** SSLV01.rtd

**COMPARISON:**

| Node | Compared result     | RSA         | AFNOR       | Difference % |
|------|---------------------|-------------|-------------|--------------|
| 200  | Displacement UX (m) | 1.500 e-3   | 1.500 e-3   | 0.0          |
| 214  | Displacement UX (m) | 1.500 e-3   | 1.500 e-3   | 0.0          |
| 208  | Displacement UX (m) | 1.500 e-3   | 1.500 e-3   | 0.0          |
| 138  | Displacement UX (m) | 1.000 e-3   | 1.000 e-3   | 0.0          |
| 76   | Displacement UX (m) | 0.500 e-3   | 0.500 e-3   | 0.0          |
| 200  | Displacement UZ (m) | -0.1497 e-3 | -0.1500 e-3 | 0.200        |
| 138  | Displacement UZ (m) | -0.1497 e-3 | -0.1500 e-3 | 0.200        |
| 76   | Displacement UZ (m) | -0.1497 e-3 | -0.1500 e-3 | 0.200        |

**CONCLUSIONS:**

Excellent agreement of results.

This test has been carried out with values of the Poisson's coefficient ranging from 0.3 to 0.499. The relation between  $-(w_a/R)/(u_a/L)$  is always equal to Poisson's coefficient.

## VERIFICATION EXAMPLE

### Uniform compression of a solid sphere - SSLV02

**Name of the test :**

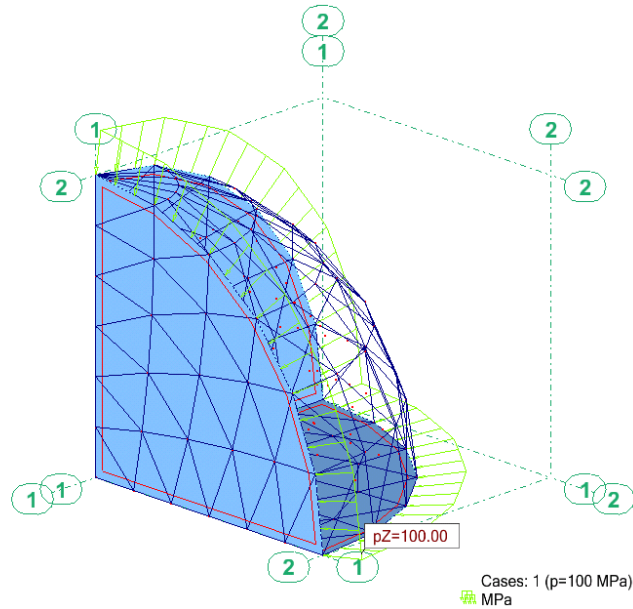
SSLV 02

**Reference :**

AFNOR

**Specification:** Solid sphere - Pressure.

**GEOMETRY :**



**DATA FILE:** SSLV02.rtd

**COMPARISON:**

| Node | Compared result            | RSA      | AFNOR    | Difference % |
|------|----------------------------|----------|----------|--------------|
| 1    | Displacement UX (m)        | -0.2 e-3 | -0.2 e-3 | 0.0          |
| 39   | Displacement UY (m)        | -0.2 e-3 | -0.2 e-3 | 0.0          |
| 14   | Displacement UZ (m)        | -0.2 e-3 | -0.2 e-3 | 0.0          |
| 1    | Stress $\sigma_{xx}$ [MPa] | - 100    | - 100    | 0.0          |
| 39   | Stress $\sigma_{yy}$ [MPa] | - 100    | - 100    | 0.0          |
| 14   | Stress $\sigma_{zz}$ [MPa] | - 100    | - 100    | 0.0          |
| 53   | Displacement UX (m)        | -0.1 e-3 | -0.1 e-3 | 0.0          |
| 61   | Displacement UY (m)        | -0.1 e-3 | -0.1 e-3 | 0.0          |
| 82   | Displacement UZ (m)        | -0.1 e-3 | -0.1 e-3 | 0.0          |
| 53   | Stress $\sigma_{xx}$ [MPa] | - 100    | - 100    | 0.0          |
| 61   | Stress $\sigma_{yy}$ [MPa] | - 100    | - 100    | 0.0          |
| 82   | Stress $\sigma_{zz}$ [MPa] | - 100    | - 100    | 0.0          |

**CONCLUSION:**

Exact agreement of results.

## VERIFICATION EXAMPLE

### Tension of a rectangular prism due to self weight - SSLV07

**Name of the test :**

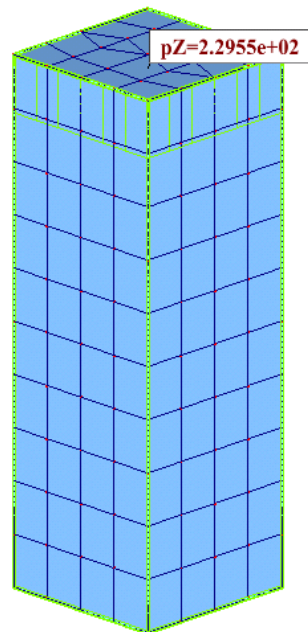
SSLV07

**Reference :**

AFNOR

**Specification:** Solid bar - Tension/compression - Poisson's coefficient.

**GEOMETRY :**



/

**DATA FILE:** SSLV07.rtd

**COMPARISON:**

| Node       | Compared result                          | RSA      | AFNOR    | Difference % |
|------------|--|----------|----------|--------------|
| 21         | Displacement UZ (m)                      | -1.72e-6 | -1.72e-6 | 0.0          |
| (21, 7)    | $\Delta$ displ. UZ (m) $w_{21}-w_7$      | 0.013e-6 | 0.014e-6 | 4.285        |
| (271, 259) | $\Delta$ displ. UX (m) $w_{271}-w_{259}$ | 0.17e-6  | 0.17 e-6 | 0.0          |
| 271        | Stress $\sigma_{zz}$ [MPa]               | 0.2191   | 0.2290   | 4.323        |
| 146        | Stress $\sigma_{zz}$ [MPa]               | 0.1147   | 0.1145   | 0.17         |

**CONCLUSION:**

Results correct.

# DYNAMIC ANALYSIS

## 1. BAR STRUCTURES

## VERIFICATION EXAMPLE

### Slender beam fixed at both ends with different inertia - SDLL03

Name of the test:

SDLL 03

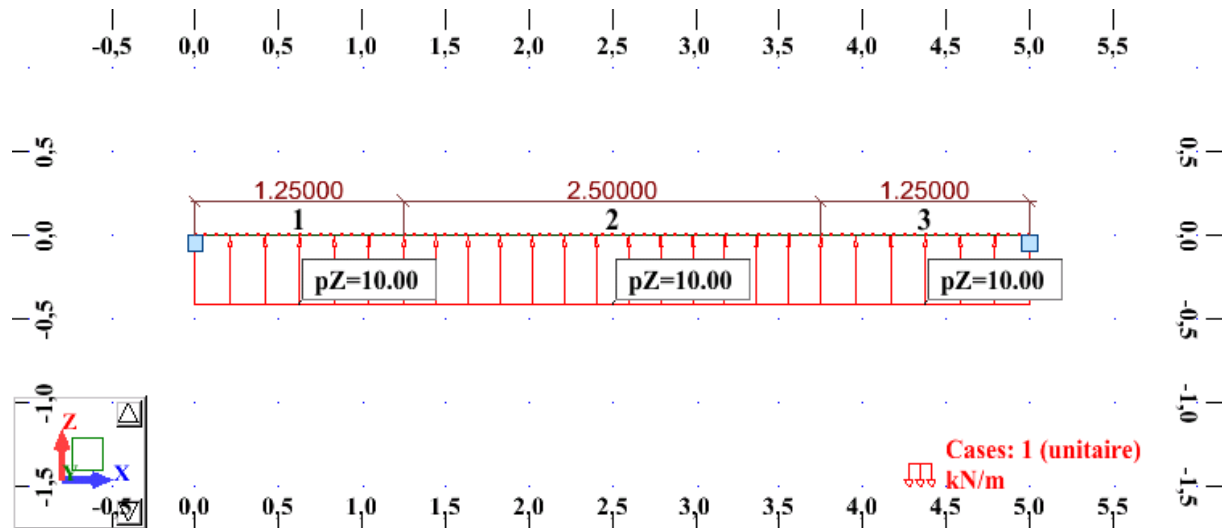
Reference:

AFNOR

Specification:

Slender beam - Eigen modes - Bending in the plane.

#### GEOMETRY:



DATA FILE:

SDLL03.rtd

#### COMPARISON:

| Node                         | Compared result              | RSA       | AFNOR     | Difference % |
|------------------------------|------------------------------|-----------|-----------|--------------|
|                              | First bending mode frequency | 62.782    | 63.009    | 0.4          |
| 56<br>X=0.4375               | Eigenvector                  | 1.428 e-2 | 1.435 e-2 | 0.5          |
| 50<br>X=0.8125               | Eigenvector                  | 3.985 e-2 | 4.002 e-2 | 0.4          |
| 2<br>X=1.25                  | Eigenvector                  | 6.872 e-2 | 6.899 e-2 | 0.4          |
| 43<br>X=1.6875               | Eigenvector                  | 8.690 e-2 | 8.922 e-2 | 2.6          |
| 37<br>X=2.0625               | Eigenvector                  | 1.005 e-1 | 1.008 e-1 | 0.3          |
| 30<br>X=2.5                  | Eigenvector                  | 1.054 e-1 | 1.057 e-1 | 0.3          |
| 31<br>t=0.0595 s<br>X=2.4375 | Vertical displacement UZ (m) | 2.231 e-3 | 2.469 e-3 | 9.6          |

#### CONCLUSION:

Results correct.



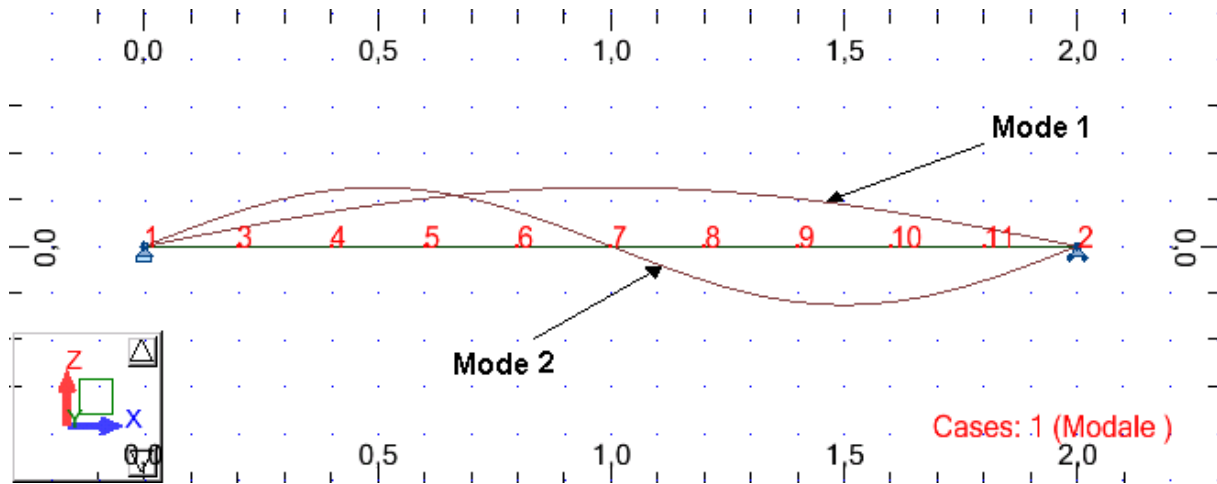


## VERIFICATION EXAMPLE

Slender beam supported at both ends subjected to axial load - SDLL05

**Name of the test:** SDLL 05  
**Reference:** AFNOR  
**Specification:** Slender beam - Bending in the plane - Eigen modes -Initial stress.

**GEOMETRY:**



**DATA FILE:** SDLL05.rtd

**COMPARISON:**

| Case       | Frequency | RSA     | AFNOR   | Difference % |
|------------|-----------|---------|---------|--------------|
| Fx=0       | Bending 1 | 28.694  | 28.702  | 0.027        |
| Fx=0       | Bending 2 | 114.701 | 114.807 | 0.093        |
| Fx=1 e+5 N | Bending 1 | 22.428  | 22.434  | 0.026        |
| Fx=1 e+5 N | Bending 2 | 108.981 | 109.080 | 0.091        |

**CONCLUSION:**

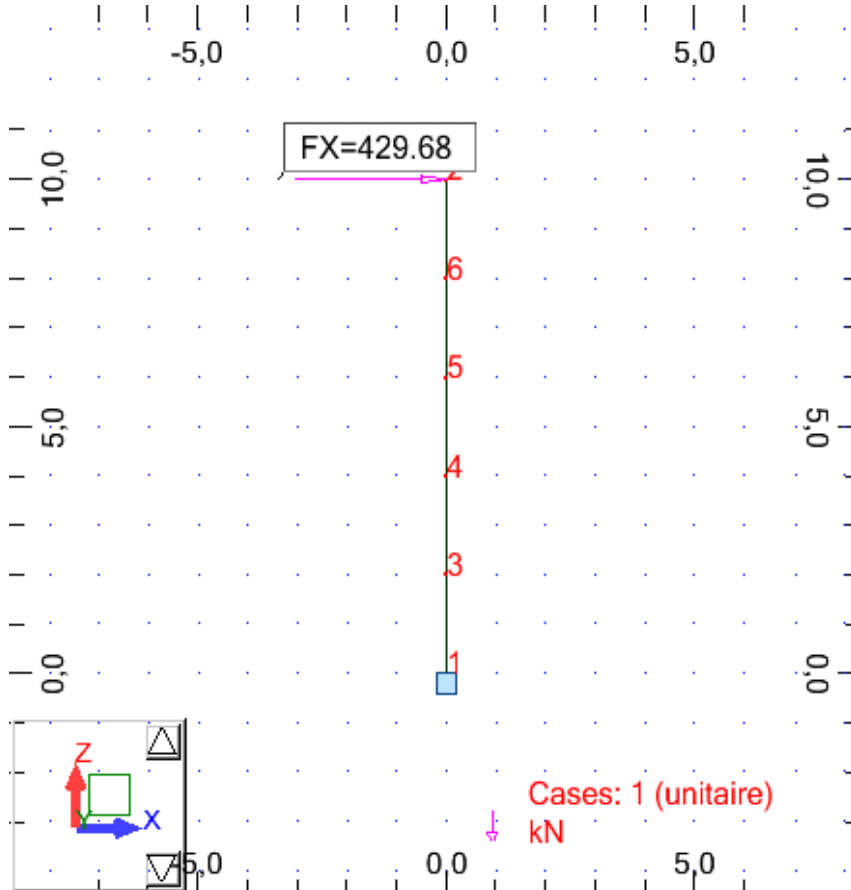
Excellent agreement of results.

**VERIFICATION EXAMPLE**

Transient analysis of a cantilever under acceleration or imposed load - SDLL 06

**Name of the test:** SDLL 06  
**Reference:** AFNOR  
**Specification:** Slender beam - Time history analysis - Plane bending - Imposed force - Imposed acceleration - Modal damping.

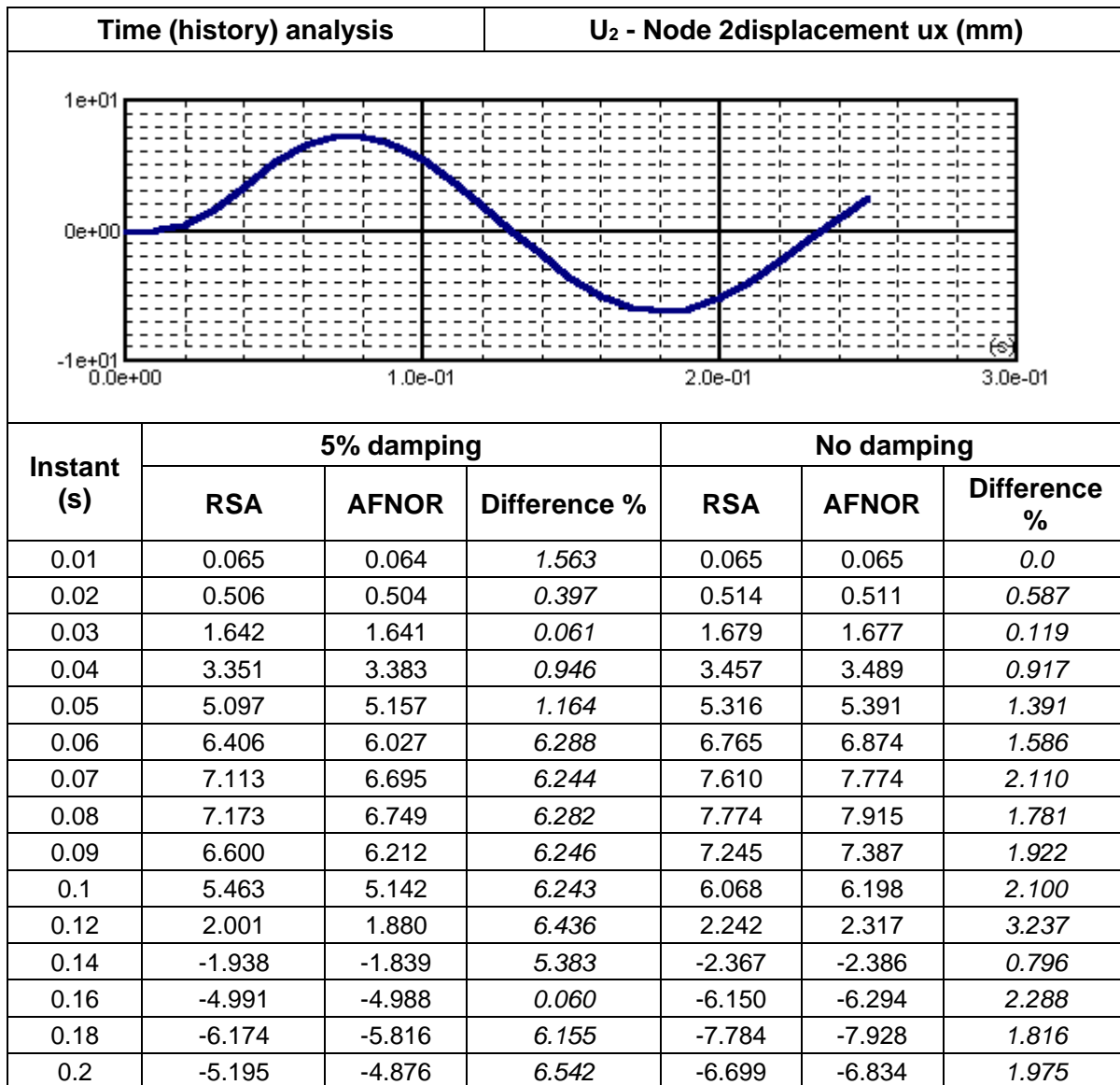
**GEOMETRY :**



**DATA FILE:** SDLL 06.rtd

**COMPARISON:**

| Modal analysis |                | RSA   | AFNOR | Difference % |
|----------------|----------------|-------|-------|--------------|
| Mode 1         | Frequency (Hz) | 4.774 | 4.774 | 0.0          |

**CONCLUSION:**

Results correct.

## VERIFICATION EXAMPLE

Slender beam supported at both ends subjected to moving load with constant speed-  
SDLL 07

**Name of the test:**

SDLL 07

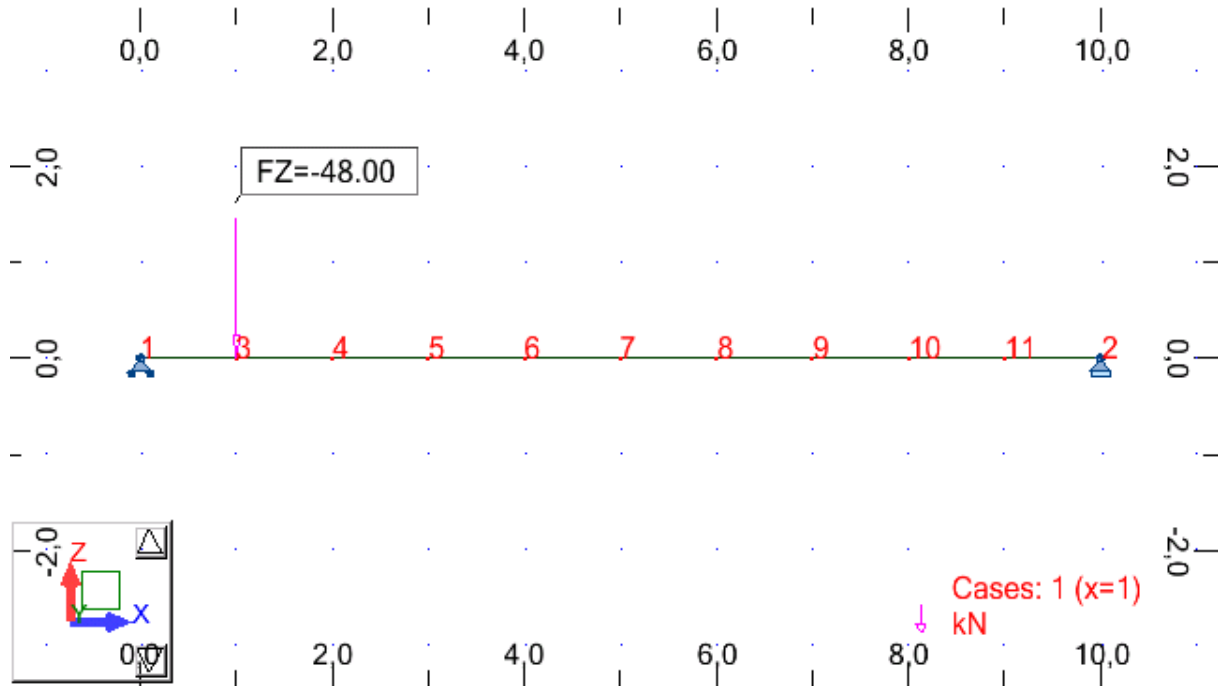
**Reference:**

AFNOR

**Specification:**

Slender beam - Bending in the plane - Eigen modes - Static initial stress.

**GEOMETRY:**



**DATA FILE:** SDLL07.rtd

**COMPARISON:**

| Instant | Compared result (node 7) | RSA      | AFNOR    | Difference % |
|---------|--------------------------|----------|----------|--------------|
| T=0.1 s | Deflection (m)           | -0.04705 | -0.04763 | 1.217        |
| T=0.2 s | Deflection (m)           | -0.3206  | -0.3235  | 0.890        |
| T=0.5 s | Deflection (m)           | -1.4254  | -1.4371  | 0.814        |

**CONCLUSION:**

Very good agreement of results.

## VERIFICATION EXAMPLE

### Plane grillage of beams - SDLL08

**Name of the test:**

SDLL 08

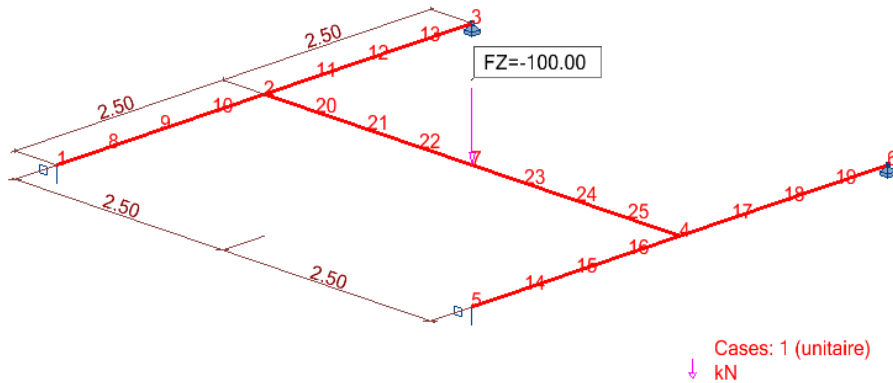
**Reference:**

AFNOR

**Specification:**

Eigen modes - Transverse bending - Imposed force.

**GEOMETRY :**



**DATA FILE:** SDLL 08.rtd

**COMPARISON:**

| Mode analysis |                        | RSA    | AFNOR  | Difference % |
|---------------|------------------------|--------|--------|--------------|
| Mode 1        | Frequency (Hz)         | 16.410 | 16.456 | 0.280        |
|               | Eigenvector w2/(w7-w2) | 1.212  | 1.213  | 0.0          |
| Mode 3        | Frequency (Hz)         | 37.941 | 38.196 | 0.668        |
|               | Eigenvector w2/(w7-w2) | -0.412 | -0.412 | 0.0          |

| Harmonic analysis |                     | RSA         | AFNOR      | Difference % |
|-------------------|---------------------|-------------|------------|--------------|
| Node 2            | Displacement UZ (m) | -100.54 e-3 | - 9.80 e-2 | 2.592        |
| Node 7            | Displacement UZ (m) | -227.74 e-3 | - 2.27 e-1 | 0.0          |

| Time history analysis |                                      | RSA         | AFNOR      | Difference % |
|-----------------------|--------------------------------------|-------------|------------|--------------|
| Node 2                | Displacement UZ (m)<br>comp.966/1001 | -98.90 e-3  | - 9.80 e-2 | 0.918        |
| Node 7                | Displacement UZ (m)<br>comp.966/1001 | -223.76 e-3 | - 2.27 e-1 | 1.427        |

**CONCLUSION:**

Very good agreement of results.

## VERIFICATION EXAMPLE

Slender cantilever fixed at both ends with variable rectangular section - SDLL09

**Name of the test:**

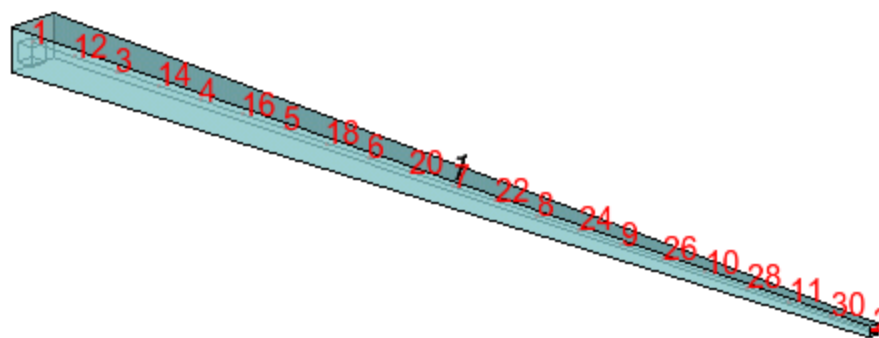
SDLL 09

**Reference:**

AFNOR

**Codification:** Eigen modes - Slender beam - Tapered section.

**GEOMETRY:**



Cases: 1 (Modale )

**DATA FILE:** SDLL09.rtd

**COMPARISON:**

| Type section | Frequency (Hz) | RSA     | AFNOR   | Difference % |
|--------------|----------------|---------|---------|--------------|
| Beta = 4     | Mode 1         | 54.19   | 54.18   | 0.02         |
|              | Mode 2         | 171.69  | 171.94  | 0.15         |
|              | Mode 3         | 383.05  | 384.40  | 0.35         |
|              | Mode 4         | 692.02  | 697.24  | 0.75         |
|              | Mode 5         | 1099.65 | 1112.28 | 1.14         |
| Beta = 5     | Mode 1         | 56.56   | 56.55   | 0.02         |
|              | Mode 2         | 175.57  | 175.79  | 0.13         |
|              | Mode 3         | 387.74  | 389.01  | 0.33         |
|              | Mode 4         | 697.21  | 702.36  | 0.73         |
|              | Mode 5         | 1105.51 | 1117.63 | 1.08         |

**CONCLUSION:**

Very good agreement of results.

## VERIFICATION EXAMPLE

Slender beam fixed at both ends with variable rectangular section - SDLL10

**Name of the test:**

SDLL 10

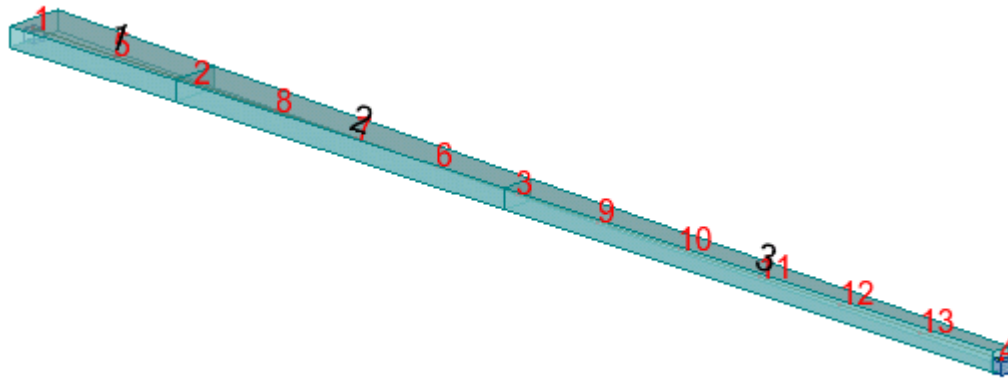
**Reference:**

AFNOR

**Specification:**

Eigen modes - Slender beam - Tapered section - Bending in the plane.

**GEOMETRY:**



Cases: 1 (Modale )

**DATA FILE:** SDLL10.rtd

**COMPARISON:**

| Frequency (Hz) | RSA      | AFNOR    | Difference % |
|----------------|----------|----------|--------------|
| Mode 1         | 145.355  | 143.303  | 1.432        |
| Mode 2         | 398.951  | 396.821  | 0.537        |
| Mode 3         | 780.806  | 779.425  | 0.177        |
| Mode 4         | 1288.503 | 1289.577 | 0.083        |

**CONCLUSION:**

Very good agreement of results.



## VERIFICATION EXAMPLE

### Ring fixed at two points - SDLL12

**Name of the test:**

SDLL 12

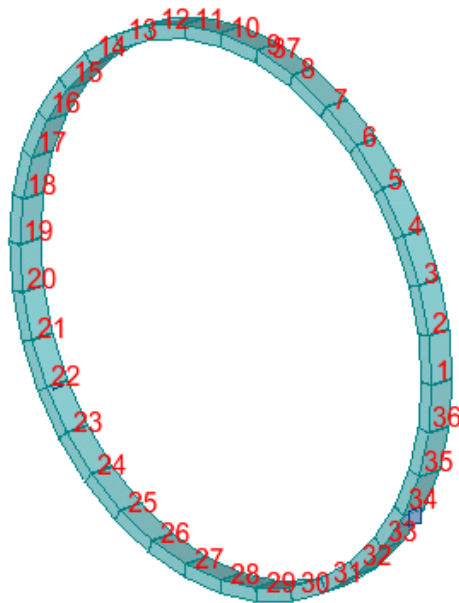
**Reference:**

AFNOR

**Specification:** Slender ring - Eigen modes - Bending in the plane.

#### GEOMETRY:

---



Cases: 1 (Modale )

**DATA FILE:** SDLL12.rtd

---

#### COMPARISON:

---

| Frequency (Hz) | RSA      | AFNOR    | Difference % |
|----------------|----------|----------|--------------|
| Mode 1         | 235.888  | 235.300  | 0.250        |
| Mode 2         | 577.053  | 575.300  | 0.305        |
| Mode 3         | 1109.262 | 1105.700 | 0.322        |
| Mode 4         | 1410.008 | 1405.600 | 0.314        |
| Mode 5         | 1755.511 | 1751.100 | 0.252        |
| Mode 6         | 2558.509 | 2557.000 | 0.059        |
| Mode 7         | 2765.514 | 2801.500 | 1.285        |

#### CONCLUSION:

---

Very good agreement of results.

## VERIFICATION EXAMPLE

### Ring with flexible support at external point - SDLL13

**Name of the test:**

SDLL 13

**Reference:**

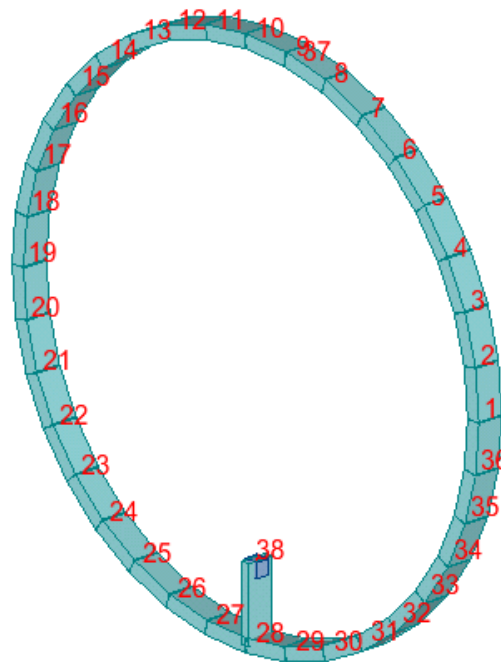
AFNOR

**Specification:**

Slender ring - Eigen modes - Bending in the plane.

#### GEOMETRY:

---



Cases: 1 (Modale )

**DATA FILE:**

SDLL13.rtd

---

#### COMPARISON:

---

| Frequency (Hz) | RSA      | AFNOR    | Difference % |
|----------------|----------|----------|--------------|
| Mode 1         | 28.814   | 28.800   | 0.049        |
| Mode 2         | 189.799  | 189.300  | 0.264        |
| Mode 3         | 269.497  | 268.800  | 0.259        |
| Mode 4         | 640.999  | 641.000  | 0.0          |
| Mode 5         | 684.410  | 682.000  | 0.353        |
| Mode 6         | 1065.192 | 1063.000 | 0.206        |

#### CONCLUSION:

---

Excellent agreement of results.



## VERIFICATION EXAMPLE

### Eigenmode of a thin-walled tube section - SDLL14

Name of the test:

SDLL 14

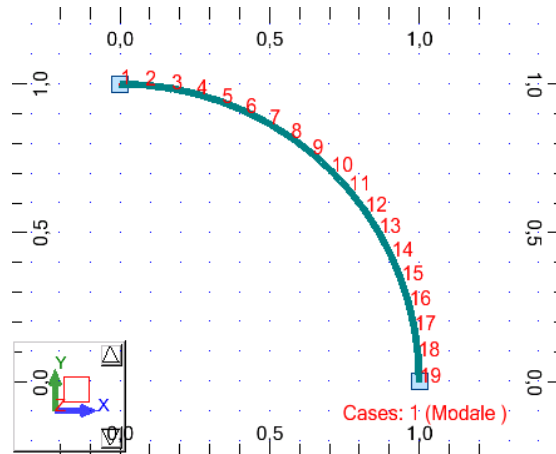
Reference:

AFNOR

Specification:

Eigen modes - Slender curved beam - Bending in the plane - Transversal bending.

#### GEOMETRY:



**DATA FILE:** SDLL14a.rtd; SDLL14b.rtd ; SDLL14c.rtd

#### COMPARISON:

| Case  |   | Frequency (Hz) | RSA     | AFNOR   | Difference % |
|-------|---|----------------|---------|---------|--------------|
| L=0   | a | Mode 1         | 44.178  | 44.230  | 0.118        |
|       |   | Mode 2         | 119.675 | 119.000 | 0.567        |
|       |   | Mode 3         | 126.058 | 125.000 | 0.846        |
|       |   | Mode 4         | 226.490 | 227.000 | 0.225        |
| L=0.6 | b | Mode 1         | 33.240  | 33.400  | 0.479        |
|       |   | Mode 2         | 94.227  | 94.000  | 0.241        |
|       |   | Mode 3         | 98.955  | 100.000 | 1.045        |
|       |   | Mode 4         | 183.372 | 180.000 | 1.873        |
| L=2   | c | Mode 1         | 17.660  | 17.900  | 1.341        |
|       |   | Mode 2         | 24.432  | 24.800  | 1.484        |
|       |   | Mode 3         | 24.949  | 25.300  | 1.387        |
|       |   | Mode 4         | 26.723  | 27.000  | 1.026        |

#### CONCLUSION:

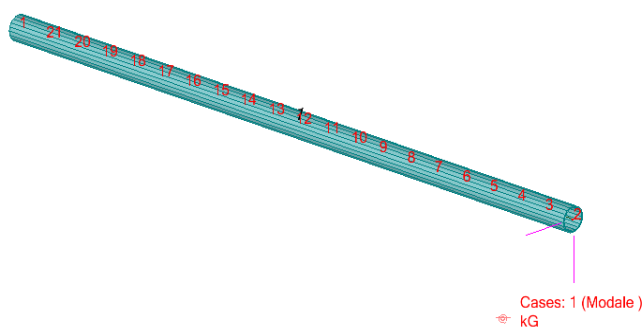
Very good agreement of results.

## VERIFICATION EXAMPLE

### Slender cantilever with mass eccentricity at the end of it - SDLL15

**Name of the test:** SDLL 15  
**Reference:** AFNOR  
**Specification:** Eigen modes - Slender beam - Bending and torsion - Bending in the plane - Transversal bending - Mass at the end of the cantilever.

#### GEOMETRY:



**DATA FILE:** SDLL15a.rtd; SDLL15b.rtd;

#### COMPARISON:

| Case |   | Frequency (Hz) | RSA     | AFNOR  | Difference % |
|------|---|----------------|---------|--------|--------------|
| Yc=0 | a | Flexion 1, 2   | 1.655   | 1.650  | 0.303        |
|      |   | Flexion 3, 4   | 16.055  | 16.070 | 0.093        |
|      |   | Flexion 5, 6   | 49.866  | 50.020 | 0.308        |
|      |   | Traction 1     | 76.473  | 76.470 | 0.0          |
|      |   | Torsion 1      | 80.469  | 80.470 | 0.0          |
|      |   | Flexion 9, 10  | 102.512 | 103.20 | 0.667        |

| Case |   | Frequency (Hz)           | RSA    | AFNOR  | Difference % |
|------|---|--------------------------|--------|--------|--------------|
| Yc=1 | b | Flexion x,z + torsion 1  | 1.636  | 1.636  | 0.0          |
|      |   | Flexion x,y + traction 2 | 1.642  | 1.642  | 0.0          |
|      |   | Flexion x,y + traction 3 | 13.446 | 13.460 | 0.104        |
|      |   | Flexion x,z + torsion 4  | 13.587 | 13.590 | 0.022        |
|      |   | Flexion x,z + torsion 5  | 28.847 | 28.900 | 0.183        |
|      |   | Flexion x,y + traction 6 | 31.929 | 31.960 | 0.097        |
|      |   | Flexion x,z + torsion 7  | 61.291 | 61.610 | 0.518        |
|      |   | Flexion x,y + traction 8 | 63.737 | 63.930 | 0.302        |

#### CONCLUSION:

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Symmetrical frame bending - SDLX01

**Name of the test:**

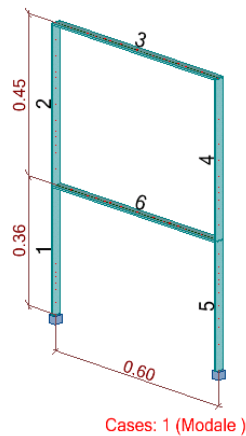
SDLX 01

**Reference:**

AFNOR

**Specification:** Slender beam - Bending in the plane - Eigen modes.

#### GEOMETRY:



**DATA FILE:** SDLX01.rtd

#### COMPARISON:

| Frequency (Hz) | RSA    | AFNOR  | Difference % |
|----------------|--------|--------|--------------|
| Mode 1         | 8.75   | 8.80   | 0.57         |
| Mode 2         | 29.35  | 29.40  | 0.17         |
| Mode 3         | 43.71  | 43.80  | 0.21         |
| Mode 4         | 56.12  | 56.30  | 0.32         |
| Mode 5         | 95.87  | 96.20  | 0.34         |
| Mode 6         | 102.37 | 102.60 | 0.22         |
| Mode 7         | 146.63 | 147.10 | 0.32         |
| Mode 8         | 174.38 | 174.80 | 0.24         |
| Mode 9         | 178.34 | 178.80 | 0.26         |
| Mode 10        | 205.56 | 206.00 | 0.21         |
| Mode 11        | 265.80 | 266.40 | 0.23         |
| Mode 12        | 319.35 | 320.00 | 0.20         |
| Mode 13        | 334.45 | 335.00 | 0.16         |

#### CONCLUSION:

Excellent agreement of results.

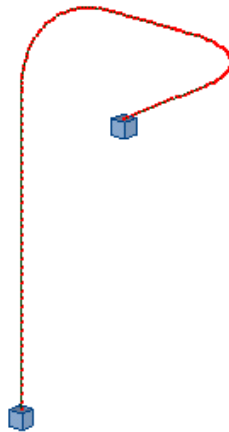
## VERIFICATION EXAMPLE

Hovgaard's problem - stress in the 3D pipe - bending - SDLX02

**Name of the test:** SDLX 02  
**Reference:** AFNOR  
**Specification:** Eigen modes - Bending in the plane - Transversal bending - Slender curved beam.

### GEOMETRY:

---



Cases: 1 (Modale )

**DATA FILE:** SDLX02.rtd

---

### COMPARISON:

---

| Frequency (Hz) | RSA    | AFNOR  | Difference % |
|----------------|--------|--------|--------------|
| Mode 1         | 10.25  | 10.18  | 0.69         |
| Mode 2         | 19.96  | 19.54  | 2.15         |
| Mode 3         | 25.08  | 25.47  | 1.53         |
| Mode 4         | 47.71  | 48.09  | 0.79         |
| Mode 5         | 52.35  | 52.86  | 0.96         |
| Mode 6         | 84.26  | 75.94  | 10.96        |
| Mode 7         | 86.51  | 80.11  | 7.99         |
| Mode 8         | 126.57 | 122.34 | 3.46         |
| Mode 9         | 130.86 | 123.15 | 6.26         |

### CONCLUSION:

---

5 first modes give correct results.





## 2. PLATES/SHELLS STRUCTURES

## VERIFICATION EXAMPLE

### Cantilever plate - SDLS01

**Name of the test:**

SDLS 01

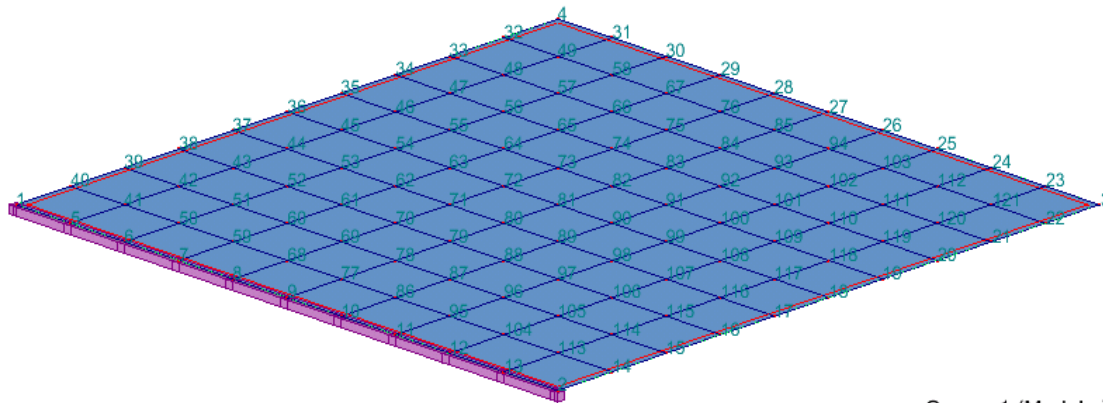
**Reference:**

AFNOR

**Specification:**

Square thin plate with one edge fixed

**GEOMETRY:**



Cases: 1 (Modale )

**DATA FILE:**

SDLS01.rtd

**COMPARISON:**

| Frequency (Hz) | RSA      | AFNOR    | Difference % |
|----------------|----------|----------|--------------|
| Mode 1         | 8.6655   | 8.7266   | 0.700        |
| Mode 2         | 21.2450  | 21.3042  | 0.278        |
| Mode 3         | 53.6890  | 53.5542  | 0.252        |
| Mode 4         | 68.5652  | 68.2984  | 0.391        |
| Mode 5         | 77.9989  | 77.7448  | 0.327        |
| Mode 6         | 137.1204 | 136.0471 | 0.789        |

**CONCLUSION:**

Excellent agreement of results.

## VERIFICATION EXAMPLE

### Lozenge - shaped thin plate with one edge fixed - SDLS02

Name of the test:

SDLS 02

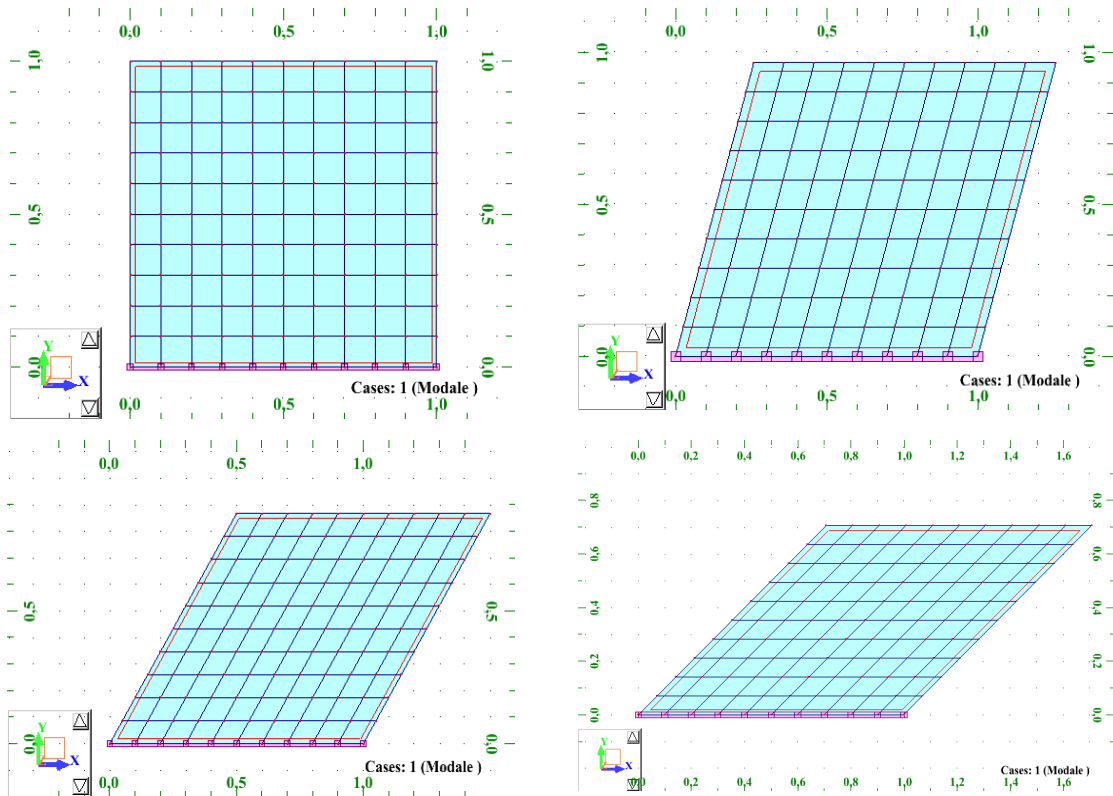
Reference:

AFNOR

Specification:

Lozenge - shaped thin plate with one edge fixed

#### GEOMETRY:



DATA FILE:

SDLS02a.rtd; SDLS02b.rtd; SDLS02c.rtd; SDLS02d.rtd

#### COMPARISON:

| ALPHA [deg] | Case | Frequency (Hz) | RSA     | AFNOR   | Difference % |
|-------------|------|----------------|---------|---------|--------------|
| 0           | a    | Mode 1         | 8.6655  | 8.7266  | 0.700        |
|             |      | Mode 2         | 21.2450 | 21.3042 | 0.278        |
| 15          | b    | Mode 1         | 8.9422  | 8.9990  | 0.631        |
|             |      | Mode 2         | 21.7167 | 22.1714 | 2.051        |
| 30          | c    | Mode 1         | 9.7945  | 9.8987  | 1.053        |
|             |      | Mode 2         | 23.4749 | 25.4651 | 7.815        |
| 45          | d    | Mode 1         | 11.29   | 11.15   | 1.256        |
|             |      | Mode 2         | 27.52   | 27.00   | 1.925        |

#### CONCLUSION:

According to "Guide de ..." accuracy of AFNOR is 3%, but for higher mode numbers it can be less accurate.

## VERIFICATION EXAMPLE

### Simply supported rectangular thin plate - SDLS03

**Name of the test:**

SDLS 03

**Reference:**

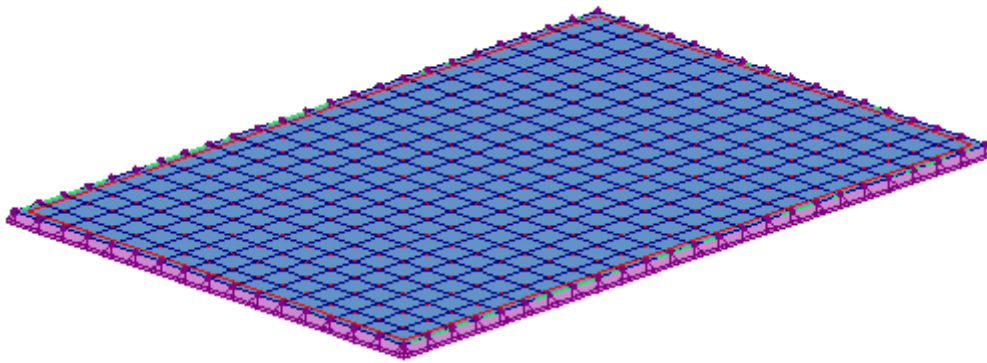
AFNOR

**Specification:**

Simply supported rectangular thin plate

#### GEOMETRY:

---



**Cases: 1 (Modale )**

**DATA FILE:**

SDLS03.rtd

---

#### COMPARISON:

---

| Frequency (Hz) | RSA    | AFNOR  | Difference % |
|----------------|--------|--------|--------------|
| Mode 1         | 35.72  | 35.63  | 0.25         |
| Mode 2         | 68.84  | 68.51  | 0.47         |
| Mode 3         | 110.85 | 109.62 | 1.12         |
| Mode 4         | 124.63 | 123.32 | 1.06         |
| Mode 5         | 143.99 | 142.51 | 1.04         |
| Mode 6         | 199.84 | 197.32 | 1.27         |

#### CONCLUSION:

---

Very good agreement of results.

## VERIFICATION EXAMPLE

### Circular plate with fixed inner edge - SDLS04

**Name of the test:**

SDLS 04

**Reference:**

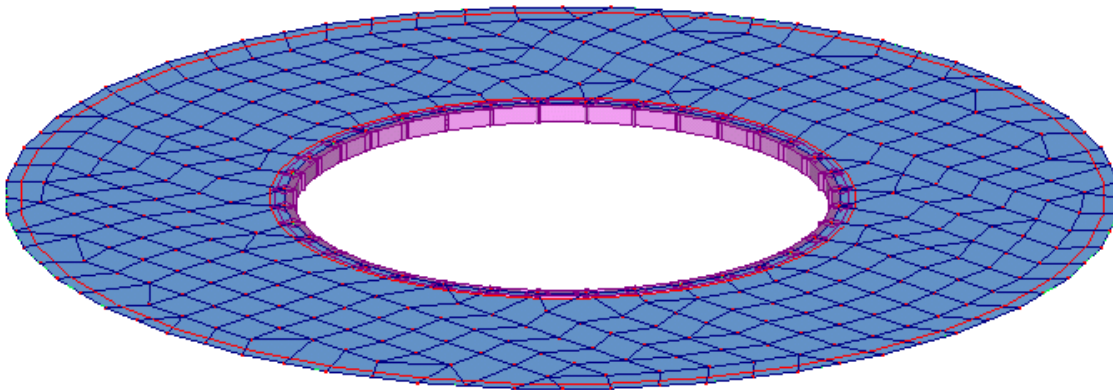
AFNOR

**Specification:**

Circular plate fixed at inner edge

**GEOMETRY:**

---



**Cases: 1 (Modale )**

**DATA FILE:**

SDLS04.rtd

---

**COMPARISON:**

---

| Mode      | Frequency $f_{ij}$ (Hz) |     | RSA            | AFNOR  | Difference % |
|-----------|-------------------------|-----|----------------|--------|--------------|
| 1         | i=0                     | j=0 | 79.48          | 79.26  | 0.28         |
| 2 and 3   | i=1                     | j=0 | 80.98, 81.12   | 81.09  | 0.14         |
| 4 and 5   | i=2                     | j=0 | 89.57, 89.62   | 89.63  | 0.06         |
| 6 and 7   | i=3                     | j=0 | 113.23         | 112.79 | 0.39         |
| 18        | i=0                     | j=1 | 526.08         | 518.85 | 1.39         |
| 19 and 20 | i=1                     | j=1 | 533.34, 540.67 | 528.61 | 2.28         |
| 21 and 22 | i=2                     | j=1 | 567.26, 570.80 | 559.09 | 2.09         |
| 23 and 24 | i=3                     | j=1 | 621.91, 622.91 | 609.7  | 2.17         |

i = number of nodal diameters

j = number of nodal circles

**CONCLUSION:**

---

Very good agreement of results.

## VERIFICATION EXAMPLE

### Compressor blade: thin shell - SDLS05

**Name of the test:**

SDLS 05

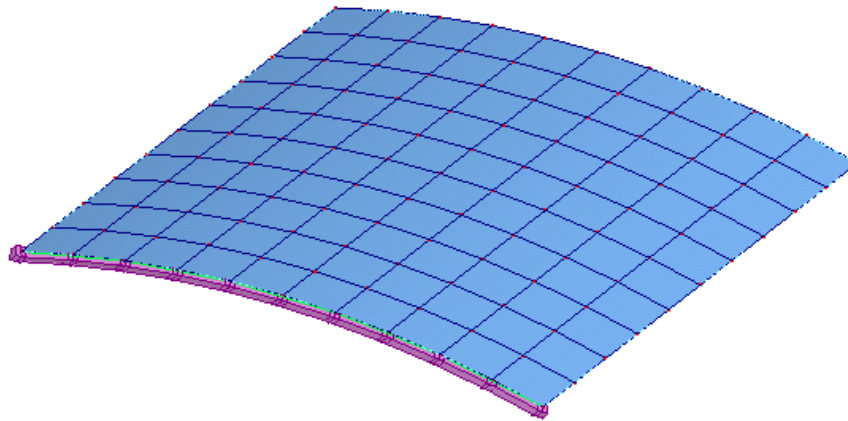
**Reference:**

AFNOR

**Specification:** Thin shell with one edge fixed

**GEOMETRY:**

---



Cases: 1 (Modale )

**DATA FILE:** SDLS05.rtd

---

**COMPARISON:**

---

| Frequency (Hz) | RSA    | AFNOR  | Difference % |
|----------------|--------|--------|--------------|
| Mode 1         | 86.12  | 85.60  | 0.61         |
| Mode 2         | 138.47 | 134.50 | 2.95         |
| Mode 3         | 250.00 | 259.00 | 3.47         |
| Mode 4         | 346.52 | 351.00 | 1.28         |
| Mode 5         | 389.68 | 395.00 | 1.35         |
| Mode 6         | 547.34 | 531.00 | 3.08         |

**CONCLUSION:**

---

According to "Guide de..." accuracy of AFNOR is 3%, but for the higher modes it is less precise.

## VERIFICATION EXAMPLE

### Modal analysis of plate - SDLS06

**Name of the test:**

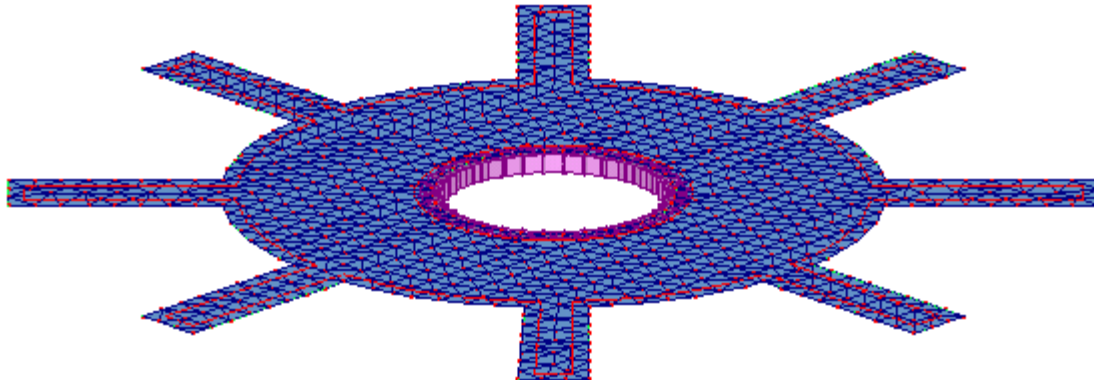
SDLS 06

**Reference:**

AFNOR

**Specification:** Eigen modes - Thin plate

#### GEOMETRY:



**Cases: 1 (Modale )**

**DATA FILE:** SDLS06.rtd

#### COMPARISON:

| Mode               | Frequency [Hz] |         | Difference % |
|--------------------|----------------|---------|--------------|
|                    | RSA            | AFNOR   |              |
| Torsion<br>Flexion | 286.91         | 295.10  | 2.85         |
|                    | 370.74         | 361.10  | 2.60         |
|                    | 399.97         | 390.50  | 2.43         |
|                    | 969.81         | 971.00  | 0.12         |
|                    | 1671.14        | 1663.00 | 0.49         |
|                    | 2178.83        | 2189.00 | 0.46         |
|                    | 2598.50        | 2627.00 | 1.08         |

#### CONCLUSION:

Good agreement of results.



# THERMOMECHANICAL ANALYSIS

## 1. BAR STRUCTURES

## VERIFICATION EXAMPLE

### Arch with 2 pinned supports - HSSL01

**Name of the test:**

HSSL 01

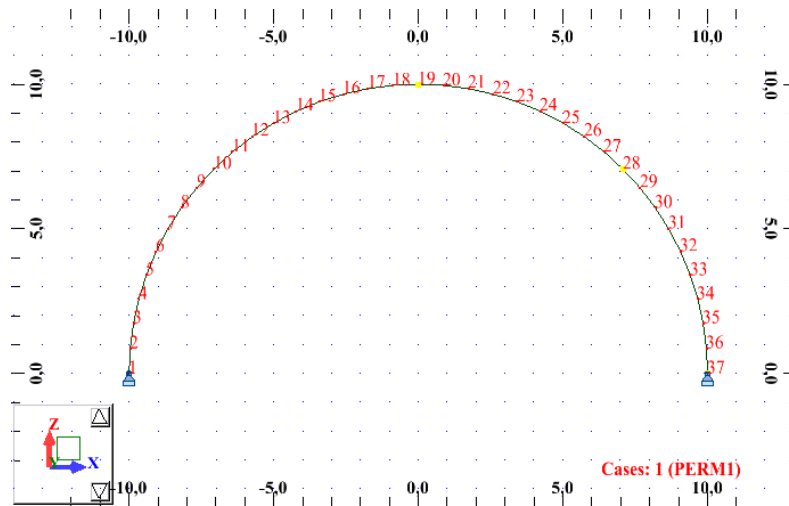
**Reference:**

AFNOR

**Specification:**

Thin-walled arch - Temperature gradient - Pinned supports.

#### GEOMETRY:



**DATA FILE:**

HSSL01.rtd

#### COMPARISON:

| Position            | Value type          | RSA       | AFNOR     | Difference % |
|---------------------|---------------------|-----------|-----------|--------------|
| Alpha=90<br>Node 37 | Bending moment(Nm)  | 0         | 0         | 0.0          |
|                     | Normal force (N)    | -209.397  | 0         |              |
|                     | Shear force (N)     | -4790.303 | -4792.000 | 0.035        |
| Alpha=45<br>Node 28 | Bending moment (Nm) | 33905.060 | 33883.000 | 0.065        |
|                     | Normal Force (N)    | -3239.121 | -3388.000 | 4.394        |
|                     | Shear force (N)     | -3535.385 | -3388.000 | 4.350        |
| Alpha=0<br>Node 19  | Bending moment (Nm) | 47948.778 | 47918.000 | 0.064        |
|                     | Normal Force(N)     | -4790.303 | -4792.000 | 0.035        |
|                     | Shear force (N)     | -209.397  | 0         |              |

#### CONCLUSIONS:

Results correct.

The results have been obtained from the average of forces from 2 bars met in a node.

Normal force (shear force) for alpha=90° (alpha=0°) is not equal to 0 because the arch consists of linear segments.

Nevertheless the value is still reliable.

## 2. PLATES/SHELLS STRUCTURES

## VERIFICATION EXAMPLE

Thin plate deformed according to spherical curve - HSL01

**Name of the test:**

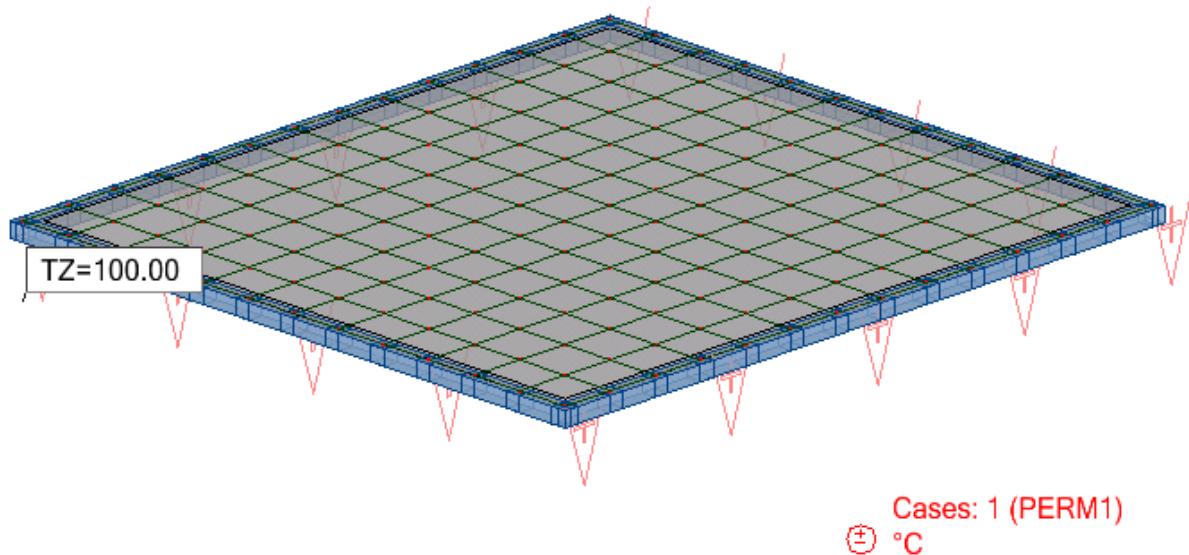
HSL01

**Reference:**

AFNOR

**Specification:** Thin plate - Thermal gradient - Fixed support.

**GEOMETRY:**



**DATA FILE** HSL01.rtd

**COMPARISON:**

| Position     | Type of the value     | RSA         | AFNOR       | Difference % |
|--------------|-----------------------|-------------|-------------|--------------|
| On the edges | Bending moment (Nm/m) | 2380.95     | 2380.95     | 0.0          |
| On the edges | Maximum stress (Pa)   | 142.857 e+6 | 142.185 e+6 | 0.47         |

**CONCLUSION:**

Excellent agreement of results.

# CONCLUSIONS

The results and accuracy achieved in verification examples confirm the quality and reliability of ROBOT Structural Analysis 2021. This state-of-the-art structural analysis and design software gives excellent accuracy within the applied solution method.