Robot Structural Analysis Professional

AUTODESK

# Comparison with NAFEMS Benchmarks

- Linear Static Benchmarks vol. 1. (Ref: LSB 1)
- Selected Benchmarks for Forced Vibration (Ref: R0016)
- Background to FE Analysis of Geometric Non-linearity Benchmarks (Ref: R0065)

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# Introduction

This report contains a range of static and dynamic benchmark tests covering a few types of behaviour encountered in structural analysis.

These examples have been taken from:

- "Linear Static Benchmarks vol.1" signed by NAFEMS as LSB1
- “Selected Benchmarks for Forced Vibration” signed by NAFEMS as R0016
- “Background to FE Analysis of Geometric Non-Linearity Benchmarks” signed by NAFEMS as R0065

**Benchmark results (signed as “NAFEMS”) are recalled and compared with results of Robot Structural Analysis Professional.**

Each problem contains the following parts:

- the name of the benchmark as used in NAFEMS manual,
- short problem description,
- scheme of the model,
- comparison between Robot Structural Analysis Professional results and reference values.

# Shell Linear Static Analysis Verification Examples

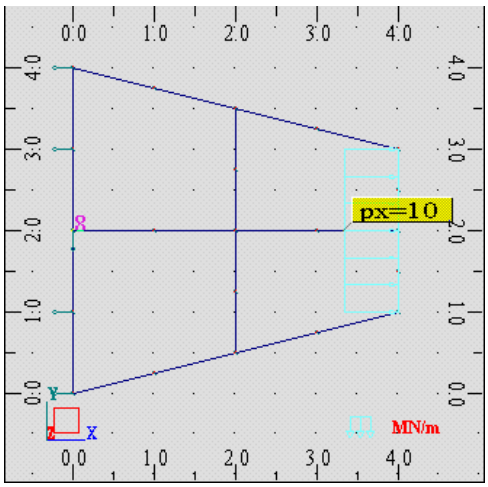
## TEST IC1: Tapered Membrane End Load

**Name of the Test:** IC1

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic membrane.

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniformly distributed horizontal load of 10 MN/m (pressure of 100 Mpa) along outer edge. |
| **Boundary Conditions:** | Nodes on X=0.0 – blocked UX, UZ, RY, node (0.0, 2.0) - fully clamped. |
| **Material Properties:** | Isotropic, E=210e3 MPa, ni=0.3 |
| **Element Type:** | Shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.TaperedMembraneEndLoad.Skyline.Nafems_IC01.rtd |

**RESULTS COMPARISON**:

| | Direct Stress Sxx at Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 61.3 | 80.64 | 31.55% |
| 4x4 | 61.3 | 68.08 | 11.06% |
| 8x8 | 61.3 | 62.61 | 2.14% |
| 16x16 | 61.3 | 61.62 | 0.52% |
| 32x32 | 61.3 | 61.41 | 0.18% |

**TARGET**: 61.3 MPa

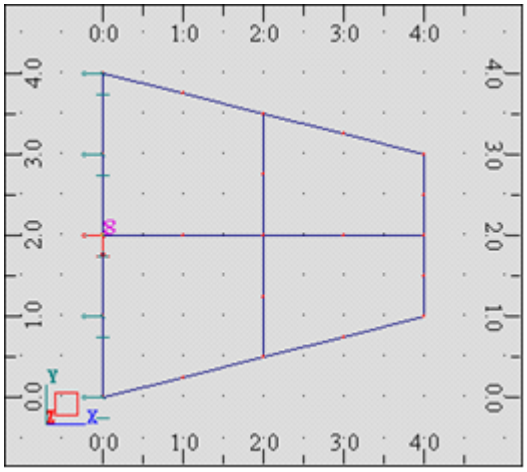
## TEST IC2: Tapered Membrane Gravity Loading

**Name of the Test:** IC2

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic membrane

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Uniform acceleration 9.81 m/s2 in global X direction (gravity).

**Boundary Conditions:** Nodes on X=0.0 – blocked UX, UZ, RY, node (0.0, 2.0) - fully clamped.

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3, p=7 MG/m3

**Element Type:** Shell 4-node quadrilaterals

**Data File:** StaticLinear.TaperedMembrane.GravityLoad.Skyline.NAFEMS_IC02.rtd

**RESULTS COMPARISON**:

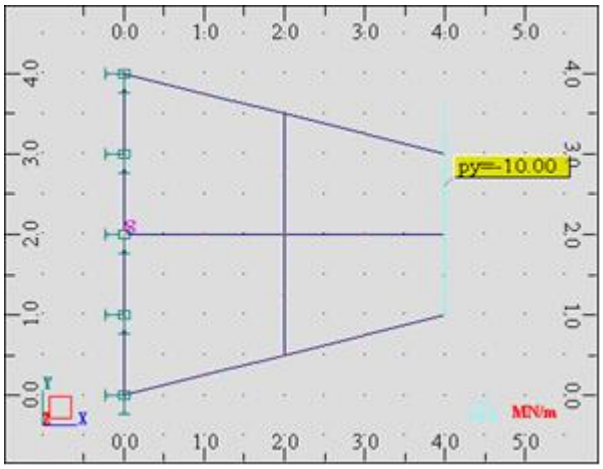
| | Direct Stress Sxx at Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 0.247 | 0.2084 | 15.63% |
| 4x4 | 0.247 | 0.229 | 7.29% |
| 8x8 | 0.247 | 0.2321 | 6.03% |
| 16x16 | 0.247 | 0.2387 | 3.36% |
| 32x32 | 0.247 | 0.2427 | 1.74% |

**TARGET**: 0.247 MPa

## TEST IC3: Tapered Membrane Edge Shear

**Name of the Test:** IC3
**Reference:** NAFEMS LSB1
**Specification:** Linear static analysis of an elastic membrane

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Uniform surface shear traction of 100 Mpa in the vertical Y- direction.
**Boundary Condition:** Edge X=0.0 – fully fixed.
**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3
**Element Type:** Shell 4-node quadrilaterals
**Data File:** StaticLinear.TaperedMembrane.EdgeShear.Skyline.NAFEMS_IC03.rtd

**RESULTS COMPARISON**:

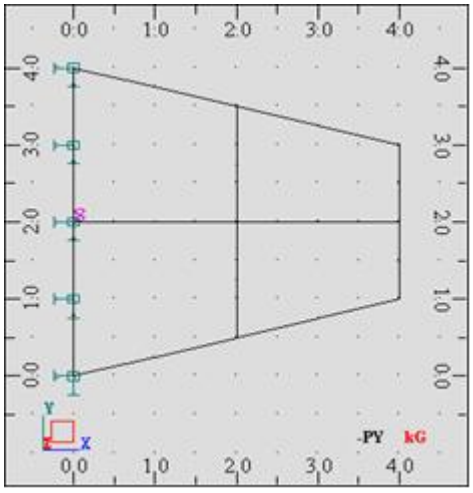
| | Direct Stress Sxy at Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 26.9 | 35.15 | 30.67% |
| 4x4 | 26.9 | 33.53 | 24.65% |
| 8x8 | 26.9 | 30.82 | 14.57% |
| 16x16 | 26.9 | 28.95 | 7.62% |
| 32x32 | 26.9 | 27.91 | 3.75% |

**TARGET**: 26.9 MPa

## TEST IC4: Tapered Cantilever Gravity Load

**Name of the Test:** IC4
**Reference:** NAFEMS LSB1
**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Uniform acceleration 9.81 m/s2 in the vertical Y direction (gravity).
**Boundary Condition:** Edge X=0.0 – fully fixed.
**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3, p=7 MG/m3
**Element Type:** Shell 4-node quadrilaterals
**Data File:** StaticLinear.TaperedCantilever.GravityLoad.Skyline.NAFEMS_IC04.rtd

**RESULTS COMPARISON**:

| | Direct Stress Sxy at Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 0.2 | 0.1304 | 34.80% |
| 4x4 | 0.2 | 0.1677 | 16.65% |
| 8x8 | 0.2 | 0.1831 | 8.45% |
| 16x16 | 0.2 | 0.1913 | 4.35% |

| | | | |
|---|---|---|---|
| 32x32 | 0.2 | 0.1953 | 2.35% |

**TARGET**: 0.2000 MPa

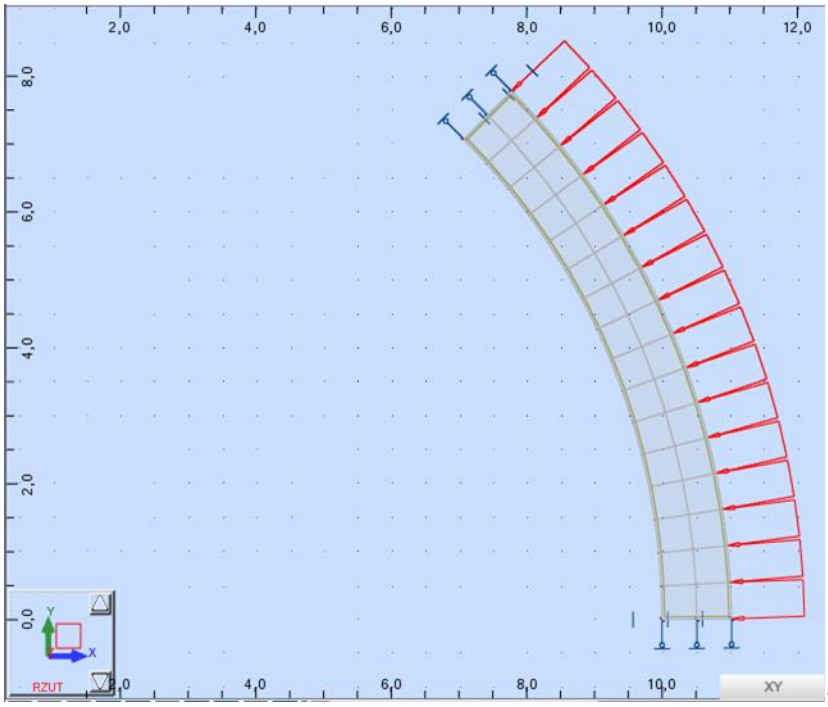
## TEST IC5: Circular Membrane Edge Pressure

**Name of the Test:** IC5

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 1 m



<Image>

**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniform inward pressure of 100MPa at outer arc edge. Inner arc edge unloaded |
| **Boundary Condition:** | Linear edges on rollers with zero hoop displacement – perpendicular directions fixed, rotations. |
| **Material Properties:** | Isotropic, E=210e3 MPa, ni=0.3 |
| **Element Type:** | shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.CircularMembrane.EdgePressure.Skyline.NAFEMS_IC05.rtd |

**RESULTS COMPARISON**:

| | Direct Stress Syy at Point No.2 (10.0, 0.0) [MPa] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 16x2 | 1150 | 1197.19 | 4.10% |

**TARGET**: 1150 MPa

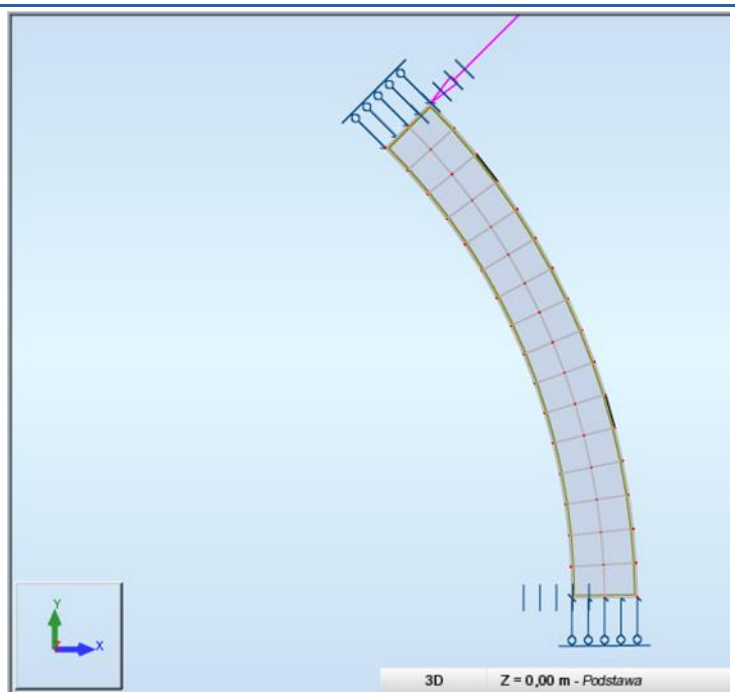
# TEST IC6: Circular Membrane Point Load

**Name of the Test:** IC6

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 1 m




**DATA DEFINITION**:

**Loading:** Point load of 5000kN radially at point B.

**Boundary Condition:** Linear edges on rollers with zero hoop displacement – perpendicular directions fixed, rotations.

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3

**Element Type:** shell 4-node quadrilaterals

**Data File:** ShellLinearStaticAnalysis.Nafems_IC06.rtd

**RESULTS COMPARISON**:

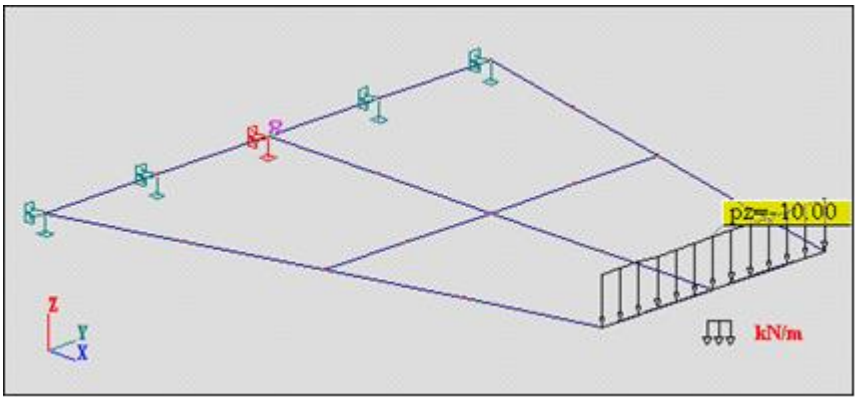
| | Direct Stress Syy at Point No.2 (10.0, 0.0) [MPa] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 16x2 | -53.2 | -61.43 | 15.47% |

**TARGET**: -53.2 MPa

## TEST IC10: Tapered Plate Edge Shear

**Name of the Test:** IC10
**Reference:** NAFEMS LSB1
**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Uniform vertical shear 10 kN/m in the Z direction along outer edge.
**Boundary Condition:** Edge X=0.0 – fully fixed.
**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3
**Element Type:** shell 4-node quadrilaterals
**Data File:** StaticLinear.TaperedPlate.EdgeShear.Skyline.NAFEMS_IC10.rtd

**RESULTS COMPARISON**:

| | Direct Stress Sxx on Top Surface Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 14.7 | 14.75 | 0.34% |
| 4x4 | 14.7 | 14.755 | 0.37% |
| 8x8 | 14.7 | 14.667 | 0.22% |
| 16x16 | 14.7 | 14.636 | 0.44% |
| 32x32 | 14.7 | 14.632 | 0.46% |

**TARGET**: 14.7 MPa

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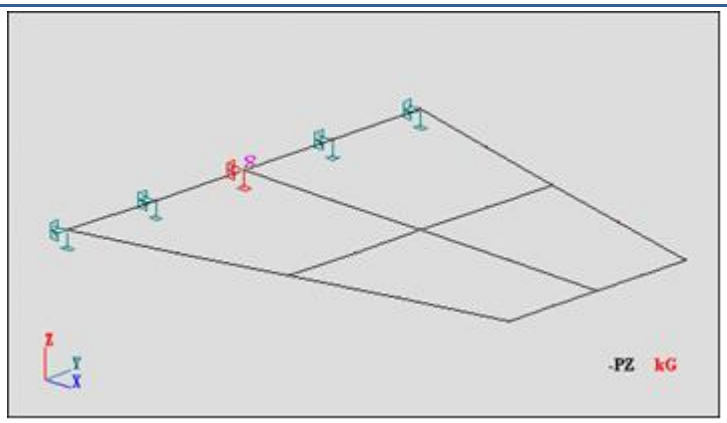
# TEST IC11: Tapered Plate Gravity Load

**Name of the Test:** IC11

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Uniform acceleration 9.81 m/s2 in the vertical Z direction (gravity).

**Boundary Condition:** Edge X=0.0 – fully fixed.

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3, p=7 MG/m3

**Element Type:** shell 4-node quadrilaterals

**Data File:** StaticLinear.TaperedPlate.GravityLoad.Skyline.NAFEMS_IC11.rtd

**RESULTS COMPARISON**:

| | Direct Stress Sxx on Top Surface Point No.8 (0.0, 2.0) | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 26 | 24.513 | 5.72% |
| 4x4 | 26 | 26.199 | 0.77% |
| 8x8 | 26 | 25.963 | 0.14% |
| 16x16 | 26 | 25.885 | 0.44% |
| 32x32 | 26 | 25.869 | 0.50% |

**TARGET**: 26 MPa

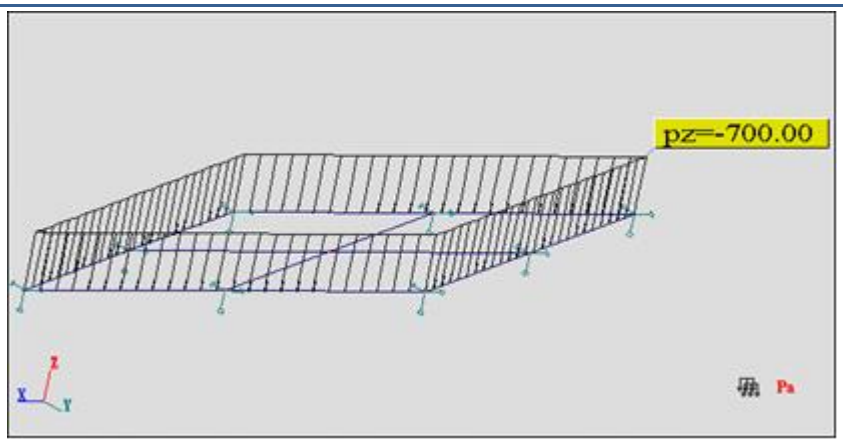
# TEST IC13: Skew Plate Normal Pressure

**Name of the Test:** IC13

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic plate

**GEOMETRY**: Thickness = 0.01 m




**DATA DEFINITION**:

**Loading:** Normal pressure –0.7 kPa in the vertical Z direction.

**Boundary Condition:** Simple supports (no z-displacement) for all edges.

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3

**Element Type:** shell 4-node quadrilaterals

**Data File:** StaticLinear.SkewPlate.NormalPressure.Skyline.NAFEMS_IC13.rtd

**RESULTS COMPARISON**:

| | Maximum Principal Stress on Lower Surface at the Plate Center | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 4x4 | 0.802 | 0.998 | 24.44% |
| 8x8 | 0.802 | 0.872 | 8.73% |
| 16x16 | 0.802 | 0.833 | 3.87% |
| 32x32 | 0.802 | 0.818 | 2.00% |

**TARGET**: 0.802 MPa

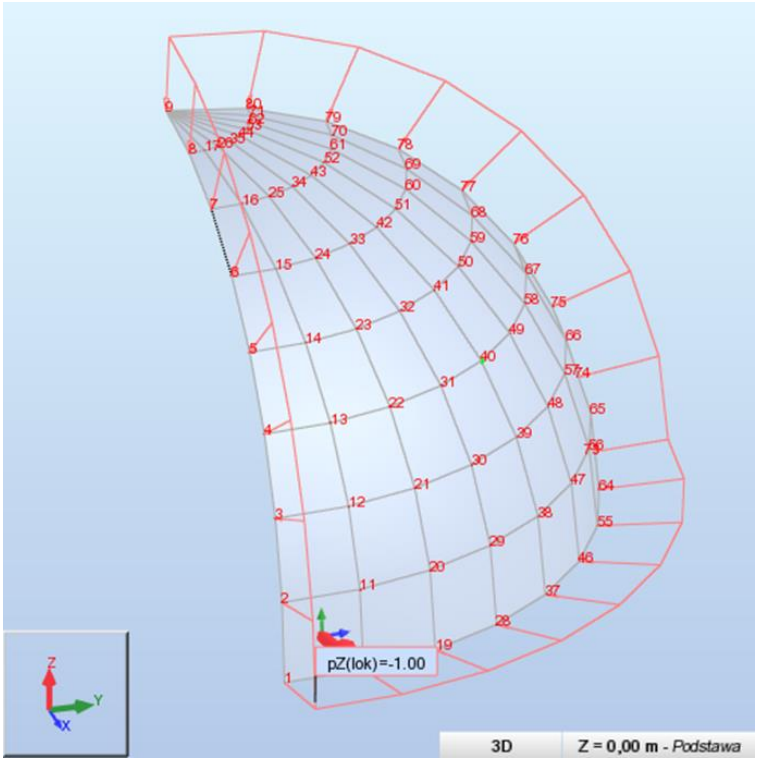
# TEST IC17: Hemisphere External Pressure

**Name of the Test:** IC17

**Reference:** NAFEMS LSB1

**Specification:** Linear elastic thin shell

**GEOMETRY**: Thickness = 0.4 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniform pressure of 1kPa directed radially inwards. |
| **Boundary Condition:** | Edge in XY, zero z-displacements, edge in YZ, zero x-displacements or rotations about y and z axes, edge in XZ, zero y-displacements or rotations about x and z axes. |
| **Material Properties:** | Isotropic, E=68250 MPa, ni=0.3 |
| **Element Type:** | shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.Hemisphere.ExternalPressure.Skyline.NAFEMS_IC17.rtd |

**RESULTS COMPARISON**:

| | Radial Displacement at Point No. 40 (5.88, 5.88, 5.56) [mm] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 8x8 | 0.001282 | 0.001262 | 1.56% |

**TARGET**: -1.282x10^-3 mm

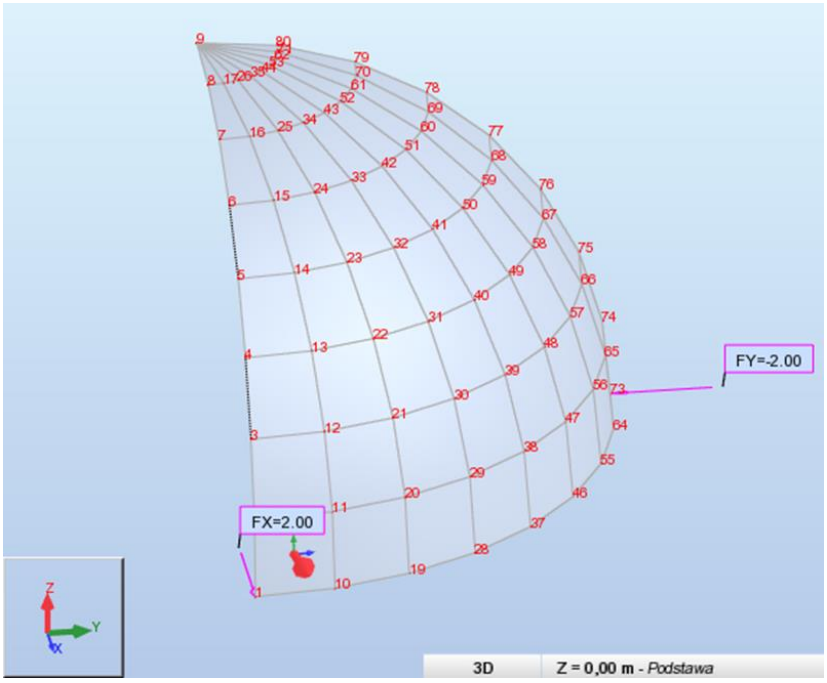
# TEST IC18: Hemisphere Point Loads

**Name of the Test:** IC18

**Reference:** NAFEMS LSB1

**Specification:** Linear elastic thin shell

**GEOMETRY**: Thickness = 0.04 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Two concentrated radial loads of 2kN directed outwards and inwards. |
| **Boundary Condition:** | Point 9(0, 0, 10) fully fixed, edge in YZ, zero x-displacements or rotations about y and z axes, edge in XZ, zero y-displacements or rotations about x and z axes. |
| **Material Properties:** | Isotropic, E=68250 MPa, ni=0.3 |
| **Element Type:** | shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.Hemisphere.PointLoad.Skyline.NAFEMS_IC18.rtd |

**RESULTS COMPARISON**:

| | x - Displacement at Point No. 1 (10, 0, 0) [mm] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |

| | | | |
|---|---|---|---|
| 8x8 | 185 | 176.7 | 4.49% |

**TARGET**: 0.185 m

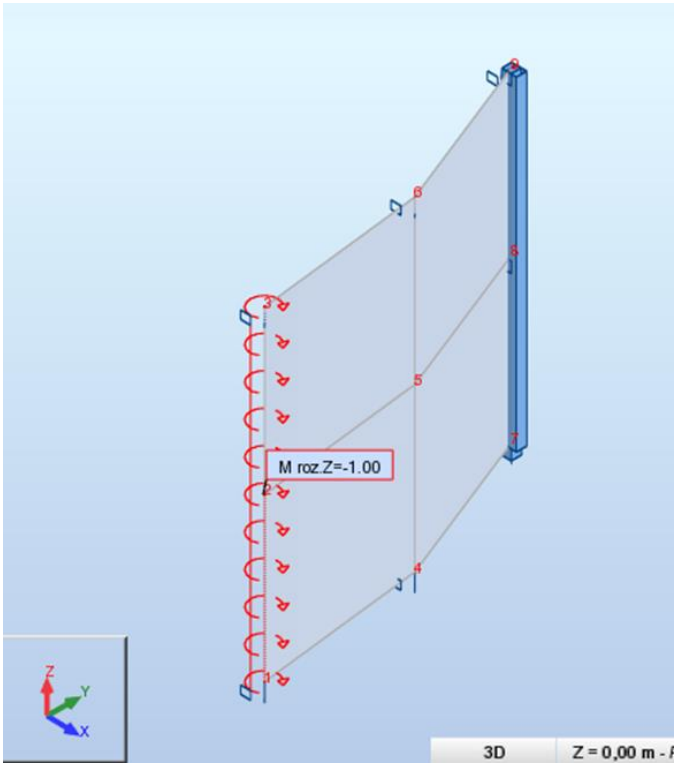
## TEST IC19: Cylindrical Shell Edge Moment

**Name of the Test:** IC19

**Reference:** NAFEMS LSB1

**Specification:** Linear elastic thin shell

**GEOMETRY**: Thickness = 0.01 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniform normal edge moment on edge DC of 1kN/m. |
| **Boundary Condition:** | Edge AB, all translations and rotations zero, edge AD and BC symmetry, z-translation and rotations about the edges are zero. |
| **Material Properties:** | Isotropic, E=210000 MPa, ni=0.3 |
| **Element Type:** | shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.Cylinder.EdgeMoment.Skyline.NAFEMS_IC19.rtd |

**RESULTS COMPARISON**:

| | Outer Layer Tangential Stress at Point No. 5 [MPa] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 60 | 60.017 | 0.03% |

**TARGET**: 60 MPa

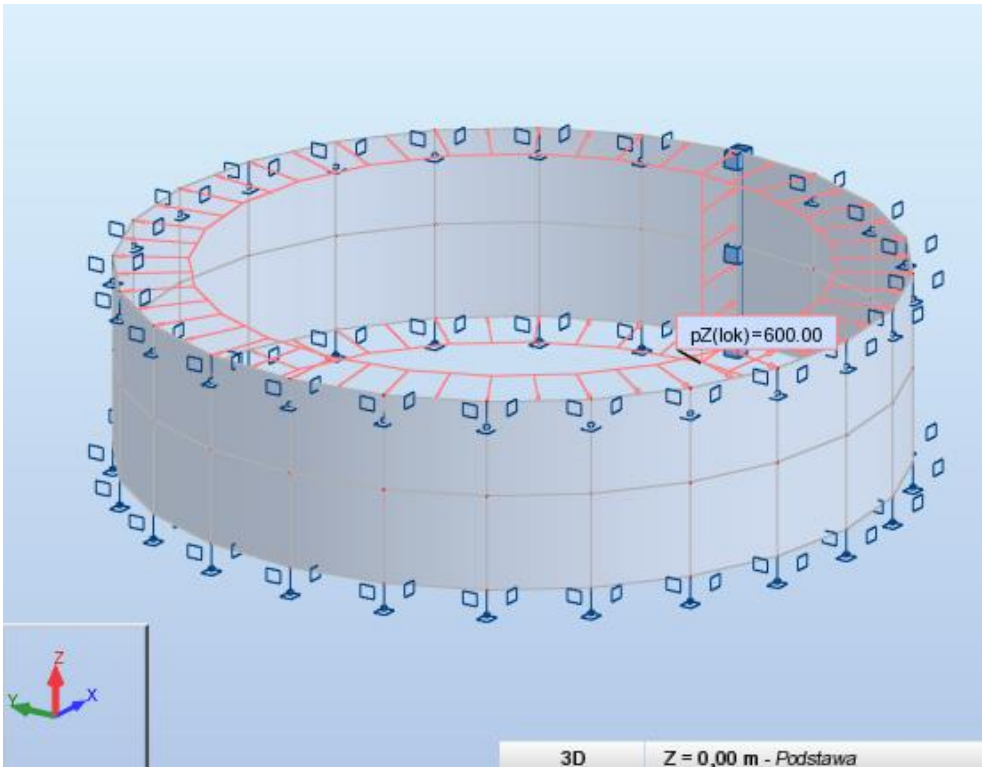
## TEST IC20: Cylindrical Shell Pressure Load

**Name of the Test:** IC20

**Reference:** NAFEMS LSB1

**Specification:** Linear elastic thin shell

**GEOMETRY**: Thickness = 0.01 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniform outward normal pressure of 0.6MPa. |
| **Boundary Condition:** | Edge AB, all translations and rotations zero, edge AD and BC symmetry, z-translation and rotations about the edges are zero. |
| **Material Properties:** | Isotropic, E=210000 MPa, ni=0.3 |
| **Element Type:** | shell 4-node quadrilaterals |
| **Data File:** | StaticLinear.Cylinder.PressureLoad.Skyline.NAFEMS_IC20.rtd |

**RESULTS COMPARISON**:

| | Outer Layer Tangential Stress at Point No. 5 [MPa] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 60 | 59.487 | 0.85% |

**TARGET**: Text

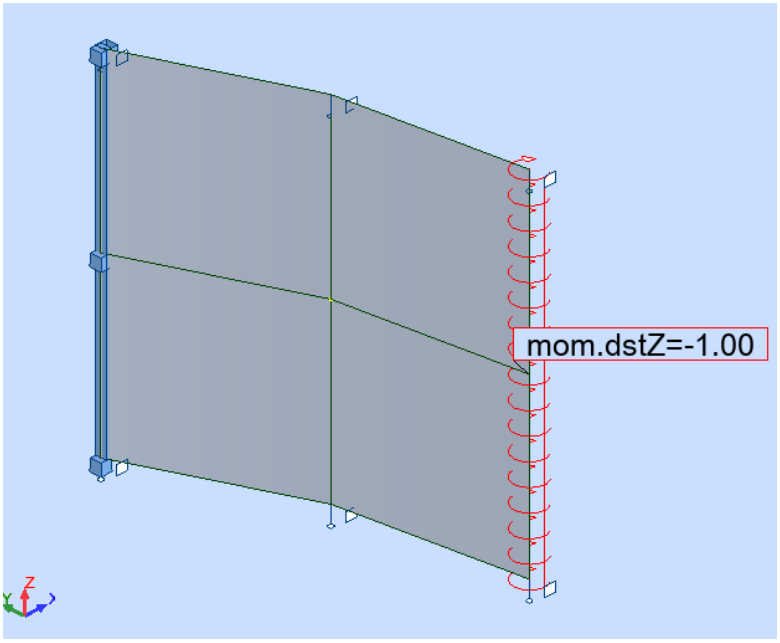
## TEST IC21: Cylindrical Shell Edge Moment

**Name of the Test:** IC21

**Reference:** NAFEMS LSB1

**Specification:** Linear elastic thin shell

**GEOMETRY**: Thickness = 0.01 m




**DATA DEFINITION**:

**Loading:** Uniform normal edge moment on edge DC of 1kN/m.

**Boundary Condition:** Edge AB, all translations and rotations zero, edge AD and BC symmetry, z-translation and rotations about the edges are zero.

**Material Properties:** Isotropic, E=210000 MPa, ni=0.3

**Element Type:** shell 4-node quadrilaterals

**Data File:** StaticLinear.Cylinder.EdgeMoment.Skyline.NAFEMS_IC21.rtd

**RESULTS COMPARISON**:

| | Outer Layer Tangential Stress at Point No. 5 [MPa] | | |
|---|---|---|---|
| **Mesh Refinement** | **TARGET** | **Robot Structural** | **Difference** |
| 2x2 | 60 | 60.017 | 0.03% |

**TARGET**: 60 MPa

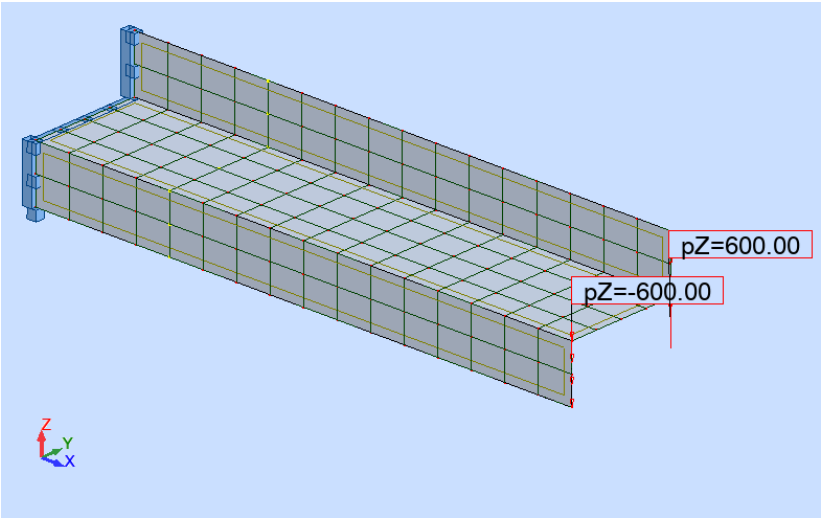
## TEST IC29: Z-Section Cantilever Torsion Bending

**Name of the Test:** IC29

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic shell

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Torque of 1.2 MNm at end x=10, by two uniformly distributes edge shears, S=0.6 MN/m at each flange.

**Boundary Condition:** At edge x=0 all displacements are zero.

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3

**Element Type:** Shell 4-node quadrilaterals

**Data File:** StaticLinear.ZSectionCantilever.Torsion.Skyline.NAFEMS_IC29.rtd

**RESULTS COMPARISON**:

| | Direct Stress Sxx at Mid Surface in 1/4 of the Length of the Beam | | | |
|---|---|---|---|---|
| **Point** | **TARGET** | **NAFEMS** | **Robot Structural** | **Difference** |
| 1 | -108.8 | -110.1 | -112.6 | 3.50% |
| 2 | -36.26 | -36.9 | -36.6 | 1.05% |
| 3 | 36.26 | 36.2 | 37.8 | 4.19% |
| 4 | 36.26 | 37.3 | 36.9 | 1.85% |
| 5 | 36.26 | 36.2 | 37.8 | 4.19% |
| 6 | -36.26 | -36.9 | -36.6 | 1.05% |
| 7 | -108.8 | -110.1 | -112.6 | 3.50% |

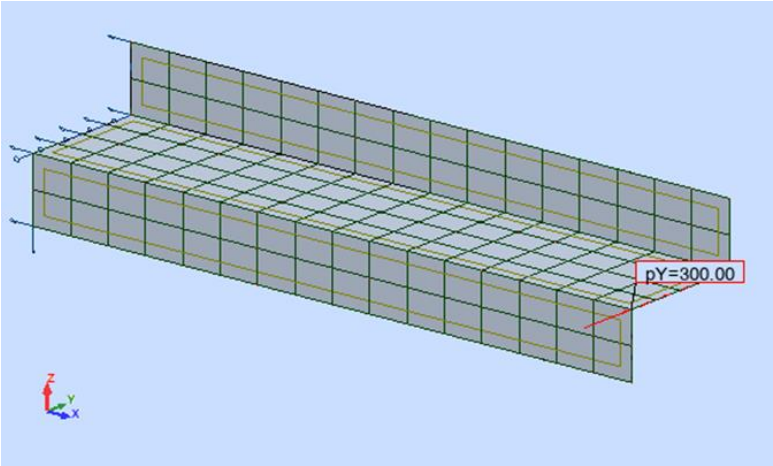
## TEST IC30: Z-Section Cantilever Beam Bending

**Name of the Test:** IC30

**Reference:** NAFEMS LSB1

**Specification:** Linear static analysis of an elastic shell

**GEOMETRY**: Thickness = 0.1 m




**DATA DEFINITION**:

**Loading:** Shear force S=0.6 MN as a uniformly distributed edge shear on the central web.

**Boundary Condition:** At edge x=0 all x-displacements are zero.
y-displacements are zero at the origin,
z-displacements are zero at the corners of the two flanges at (0,-1,-1) and (0,1,1)

**Material Properties:** Isotropic, E=210e3 MPa, ni=0.3

**Element Type:** Shell 4-node quadrilaterals

**Data File:** StaticLinear.ZSectionCantilever.Bending.Skyline.NAFEMS_IC30.rtd

**RESULTS COMPARISON**:

| | Direct Stress Sxx at Mid Surface in 1/4 of the Length of the Beam | | | |
|---|---|---|---|---|
| **Point** | **TARGET** | **NAFEMS** | **Robot Structural** | **Difference** |
| 1 | 193 | 191 | 201.36 | 4.33% |
| 2 | -96.5 | -96.7 | -97.15 | 0.67% |
| 3 | -386 | -383 | -391.23 | 1.35% |
| 4 | 0 | 0 | 0 | 0.00% |
| 5 | 386 | 383 | 391.23 | 1.35% |
| 6 | 96.5 | 96.7 | 97.15 | 0.67% |
| 7 | -193 | -191 | -201.36 | 4.33% |

# Dynamic Analysis Verification Examples

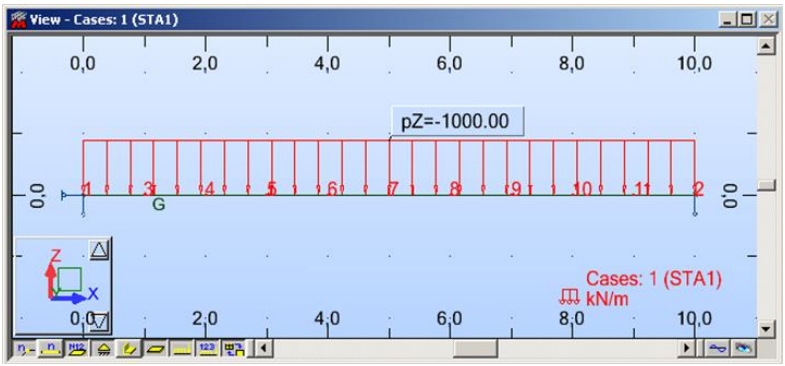
## Number 5 TESTS: Vibrations of a Deep Beam

**Name of the Tests:** 5, 5H, 5P, and 5T

**Reference:** NAFEMS R0016

**Specification:** Dynamic analysis of an elastic beam

**GEOMETRY**: Length: L = 10 m
Section: a = b = 2 m




**DATA DEFINITION**:

**Loading:** Uniform load Fo=$10^6$ [N/m] (Fo=1000 [kN/m])

**Boundary Conditions:** X=Y=Z=RX=0 (at the beginning of the beam)
Y=Z=0 (at the end of the beam)

**Material Properties:** E=200x$10^9$ N/m$^2$; ν=0,3; ρ=8000 kg/m$^3$

**Element Type:** 10 beam elements, using attribute: 'Consider shear forces in deformation calculation' (Timoshenko's Beam – deep beam).

**Data File:** DeepBeamVibration.TimeHistory.Skyline.Decomposition.NAFEMS_05.rtd

## Results Comparison of Modal Analysis (5)

| Modes | NAFEMS | Robot Structural | Difference |
|---|---|---|---|
| 1&2 | 42.65 | 42.49 | 0.38% |
| 3 | 71.2 | 71.26 | 0.08% |
| 4 | 125 | 125.11 | 0.09% |
| 5&6 | 148.15 | 143.79 | 2.94% |
| 7 | 213.61 | 215.54 | 0.90% |
| 8&9 | 283.47 | 259.36 | 8.51% |

**Output**: Frequencies [Hz]

## Results Comparison of Harmonic Forced Vibration (5H)

| Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|
| NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 13.45 | 13.43 | 0.15% | 241.9 | 242.4 | 0.21% |

**Forcing Function:** $F=Fo\sin(2\pi ft)$

**Output**: Peak Displacement [mm]
Peak Stress [MPa]

**Note**: Response at the middle node of the beam (node no 7) for the frequency f=20[Hz]

## Results Comparison of Periodic Forced Vibration (5P)

| Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|
| NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 0.951 | 0.953 | 0.21% | 17.10 | 17.39 | 1.71% |

**Forcing Function:** $F=Fo[\sin(2\pi ft)-\sin(3(2\pi ft))]$

**Output:** Peak Displacement [mm]
Peak Stress [MPa]

**Note:** Response at the middle node of the beam (node no 7) for the frequency f=20[Hz]

## Results Comparison of Impulse Forced Vibration (5T)

| Peak Displacement [mm] | | | At the Time [s] | | |
|---|---|---|---|---|---|
| NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |

| | | | | | |
|---|---|---|---|---|---|
| 1.043 | 1.047 | 0.02% | 0.0117 | 0.0117 | 0.0% |

| Peak Stress [MPa] | | | Static Displacement [mm] | | |
|---|---|---|---|---|---|
| **NAFEMS** | **Robot Structural** | **Difference** | **NAFEMS** | **Robot Structural** | **Difference** |
| 18.76 | 18.67 | 0.21% | 0.538 | 0.535 | 0.56% |

**Forcing Function:** F=Fo

**Output:** Peak Displacement [mm] and the corresponding Time [s]
Peak Stress [MPa]
Static Displacement [mm]

**Note:** Response at the middle node of the beam (node no 7)

# Number 13 TESTS: Vibrations of Simply Supported Thin Plate

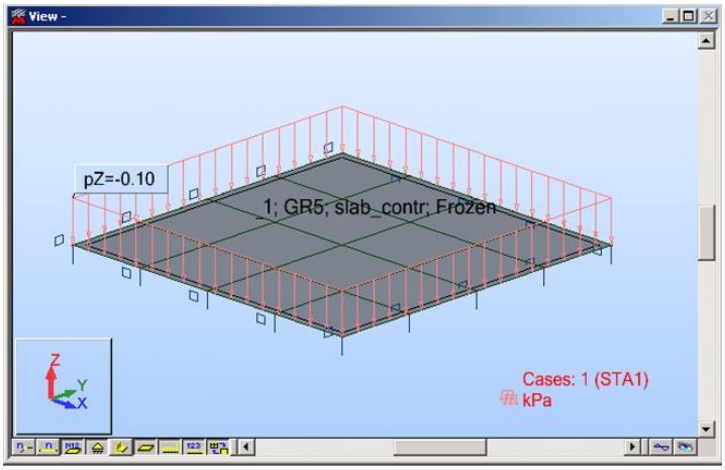
**Name of the Test:** 13, 13H, 13P, and 13T

**Reference:** NAFEMS R0016

**Specification:** Dynamic analysis of an elastic plate

**GEOMETRY**: Length: A = B = 10 m

Thickness: t = 0.05 m




**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Uniform planar load Fo=100 [N/m$^2$] (Fo=0,1 [kN/m$^2$]) |
| **Boundary Conditions:** | X=Y=RZ= 0 (at all nodes - Plate)<br>Z=0 (at all edges)<br>RX=0 (along edges X=0 & X=10 m)<br>RY=0 (along edges Y=0 & Y=10 m) |
| **Material Properties:** | E=200x10$^9$ N/m$^2$ ; $\nu$=0,3 ; $\rho$=8000 kg/m$^3$ |
| **Element Type:** | 4-node quadrilateral shell elements (three models of mesh considered: 4x4, 8x8, and 16x16 elements) |
| **Data File:** | ThinPlateVibration.TimeHistory.Skyline.Decomposition.NAFEMS_13.rtd |

## Results Comparison of Modal Analysis (13)

| Modes | NAFEMS | Robot Structural Meshing 4x4 | Difference |
|---|---|---|---|
| 1 | 2.377 | 2.512 | 5.68% |
| 2&3 | 5.942 | 7.0713 | 19.01% |
| 4 | 9.507 | 11.7394 | 23.48% |
| 5&6 | 11.884 | 16.5599 | 39.35% |
| 7&8 | 15.449 | 21.3135 | 37.96% |

| Modes | NAFEMS | Robot Structural Meshing 8x8 | Difference |
|---|---|---|---|
| 1 | 2.377 | 2.41 | 1.39% |
| 2&3 | 5.942 | 6.2159 | 4.61% |
| 4 | 9.507 | 10.0467 | 5.68% |
| 5&6 | 11.884 | 13.2011 | 11.08% |
| 7&8 | 15.449 | 17.0735 | 10.52% |

| Modes | NAFEMS | Robot Structural Meshing 16x16 | Difference |
|---|---|---|---|
| 1 | 2.377 | 2.3849 | 0.33% |
| 2&3 | 5.942 | 6.0086 | 1.12% |
| 4 | 9.507 | 9.6378 | 1.38% |
| 5&6 | 11.884 | 12.2013 | 2.67% |
| 7&8 | 15.449 | 15.8396 | 2.53% |

**Output**: Frequencies [Hz]

## Results Comparison of Harmonic Forced Vibration (13H)

| Meshing | Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|---|
| | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 45.42 | 44.66 | 1.67% | 30.03 | 33.20 | 10.56% |
| 8x8 | 45.42 | 45.11 | 0.68% | 30.03 | 32.23 | 7.33% |
| 16x16 | 45.42 | 45.11 | 0.68% | 30.03 | 31.90 | 6.23% |

**Forcing Function:** $F=Fosin(2\pi ft)$

**Output:** Peak Displacement [mm]
Peak Stress [MPa]

**Note:** Response at the center of the plate (node no 1) for the 1st mode frequency f=2.512[Hz] (4x4); f=2.410[Hz] (8x8); f=2.385[Hz] (16x16)

## Results Comparison of Periodic Forced Vibration (13P)

| Meshing | Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|---|
| | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 2.863 | 3.069 | 7.2% | 2.018 | 2.316 | 14.7% |
| 8x8 | 2.863 | 2.916 | 1.87% | 2.018 | 2.124 | 5.25% |
| 16x16 | 2.863 | 2.884 | 0.73% | 2.018 | 2.080 | 3.07% |

**Forcing Function**: $F=Fo[\sin(2\pi ft)-\sin(3(2\pi ft))]$

**Output:** Peak Displacement [mm]
Peak Stress [MPa]

**Note:** Response at the center of the plate (node no 1) for frequency f=1.2[Hz]

## Results Comparison of Impulse Forced Vibration (13T)

| Meshing | Peak Displacement [mm] | | | At the Time [s] | | |
|---|---|---|---|---|---|---|
| | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 3.523 | 3.447 | 2.16% | 0.210 | 0.200 | 4.76% |
| 8x8 | 3.523 | 3.476 | 1.33% | 0.210 | 0.210 | 0.0% |
| 16x16 | 3.523 | 3.447 | 2.16% | 0.210 | 0.210 | 0.0% |

| Meshing | Peak Stress [MPa] | | | Static Displacement [mm] | | |
|---|---|---|---|---|---|---|
| | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 2.484 | 2.502 | 0,72% | 1.817 | 1.767 | 2.75% |
| 8x8 | 2.484 | 2.441 | 1.73% | 1.817 | 1.774 | 2.37% |
| 16x16 | 2.484 | 2.344 | 5.68% | 1.817 | 1.774 | 2.37% |

**Forcing Function**: F=Fo

**Output**: Peak Displacement [mm] and the corresponding Time [s]
Peak Stress [MPa]
Static Displacement [mm]

**Note**: Response at the center of the plate (node no 1)

# Number 21 TESTS: Vibrations of Simply Supported Thick Plate

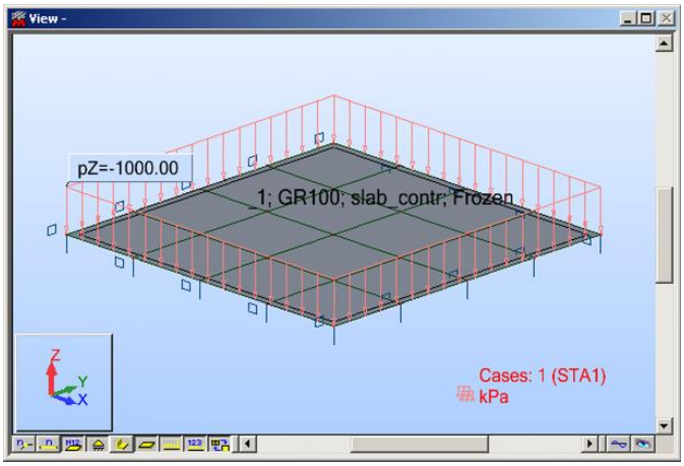
**Name of the Test:** 21, 21H, 21P, and 21T

**Reference:** NAFEMS R0016

**Specification:** Dynamic analysis of an elastic plate

**GEOMETRY**: Length: A = B = 10 m

Thickness: t = 1.0 m




**DATA DEFINITION**:

**Loading:** Uniform planar load $Fo=10^6$ [$N/m^2$] ($Fo=1000$ [$kN/m^2$])

**Boundary Conditions:**
X=Y=RZ=0 (at all nodes - Plate)
Z=0 (at all edges)
RX=0 (along edges X=0 & X=10 m)
RY=0 (along edges Y=0 & Y=10 m)

**Material Properties:** $E=200x10^9$ $N/m^2$ ; $\nu=0,3$ ; $\rho=8000$ $kg/m^3$

**Element Type:** 4-node quadrilateral shell elements (three models of mesh considered: 4x4, 8x8, and 16x16 elements).

**Data File:** ThickPlateVibration.TimeHistory.Skyline.Decomposition.NAFEMS_21.rtd

## Results Comparison of Modal Analysis (21)

| Modes | NAFEMS | Robot Structural Meshing 4x4 | Difference |
|---|---|---|---|
| 1 | 45.897 | 48.69 | 6.09% |
| 2&3 | 109.44 | 130.15 | 18.92% |
| 4 | 167.89 | 207.75 | 23.74% |
| 5&6 | 204.51 | 280.50 | 37.16% |
| 7&8 | 256.50 | 351.32 | 36.97% |

| Modes | NAFEMS | Robot Structural Meshing 8x8 | Difference |
|---|---|---|---|
| 1 | 45.897 | 46.66 | 1.66% |
| 2&3 | 109.44 | 114.85 | 4.94% |
| 4 | 167.89 | 178.54 | 6.34% |
| 5&6 | 204.51 | 226.70 | 10.85% |
| 7&8 | 256.50 | 284.25 | 10.82% |

| Modes | NAFEMS | Robot Structural Meshing 16x16 | Difference |
|---|---|---|---|
| 1 | 45.897 | 46.14 | 0.53% |
| 2&3 | 109.44 | 111.05 | 1.47% |
| 4 | 167.89 | 171.14 | 1.94% |
| 5&6 | 204.51 | 210.78 | 3.07% |
| 7&8 | 256.50 | 264.62 | 3.17% |

**Output**: Frequencies [Hz]

## Results Comparison of Harmonic Forced Vibration (21H)

| Meshing | Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|---|
| | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 45.42 | 44.66 | 1.67% | 30.03 | 33.20 | 10.56% |
| 8x8 | 45.42 | 45.11 | 0.68% | 30.03 | 32.23 | 7.33% |
| 16x16 | 45.42 | 45.11 | 0.68% | 30.03 | 31.90 | 6.23% |

**Forcing Function:** $F=Fo\sin(2\pi ft)$

**Output:** Peak Displacement [mm]
Peak Stress [MPa]

**Note:** Response at the center of the plate (node no 1) for the 1st mode frequency f=48.6925[Hz] (4x4); f=46.663[Hz] (8x8); f=46.132[Hz] (16x16)

## Results Comparison of Periodic Forced Vibration (21P)

| | Peak Displacement [mm] | | | Peak Stress [MPa] | | |
|---|---|---|---|---|---|---|
| Meshing | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 4.929 | 6.148 | 24.73% | 67.67 | 91.51 | 35.23% |
| 8x8 | 4.929 | 5.335 | 8.24% | 67.67 | 75.18 | 11.10% |
| 16x16 | 4.929 | 5.150 | 4.48% | 67.67 | 70.94 | 4.77% |

**Forcing Function**: $F=Fo[\sin(2\pi ft)-\sin(3(2\pi ft))]$

**Output:** Peak Displacement [mm]
Peak Stress [MPa]

**Note:** Response at the center of the plate (node no 1) for frequency f=20[Hz]

## Results Comparison of Impulse Forced Vibration (21T)

| | Peak Displacement [mm] | | | At the Time [s] | | |
|---|---|---|---|---|---|---|
| Meshing | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 4.524 | 4.54 | 0.35% | 0.0108 | 0.0104 | 3.7% |
| 8x8 | 4.524 | 4.508 | 0.35% | 0.0108 | 0.0105 | 2.78% |
| 16x16 | 4.524 | 4.573 | 1.11% | 0.0108 | 0.0105 | 2.78% |

| | Peak Stress [MPa] | | | Static Displacement [mm] | | |
|---|---|---|---|---|---|---|
| Meshing | NAFEMS | Robot Structural | Difference | NAFEMS | Robot Structural | Difference |
| 4x4 | 62.11 | 66.04 | 6.33% | 2.333 | 2.307 | 0.99% |
| 8x8 | 62.11 | 60.80 | 2.11% | 2.333 | 2.320 | 0.43% |
| 16x16 | 62.11 | 61.56 | 0.89% | 2.333 | 2.326 | 0.17% |

**Forcing Function**: F=Fo

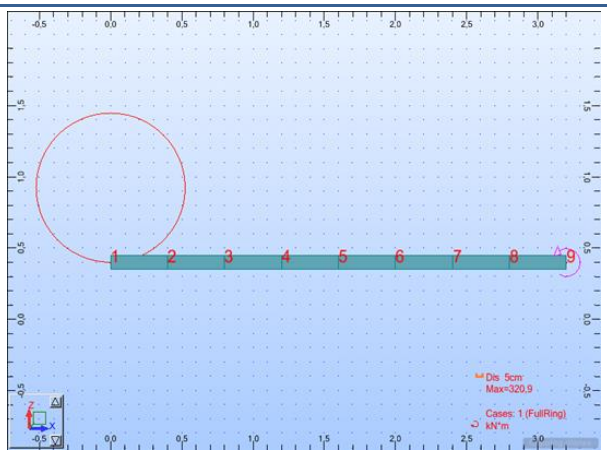
**Output**: Peak Displacement [mm] and the corresponding Time [s]
Peak Stress [MPa]
Static Displacement [mm]

**Note**: Response at the center of the plate (node no 1)

# Large Rotation and Displacement Verification Examples

## TEST GNL-5: Large rotations and displacements of a straight cantilever

| | |
|---|---|
| **Name of the Test:** | GNL-5 |
| **Reference:** | NAFEMS R0065 |
| **Specification:** | Geometric nonlinearity |
| **GEOMETRY**: | Length: 3,2 m<br>Cross Section: rectangular 0.1x0.1 m |

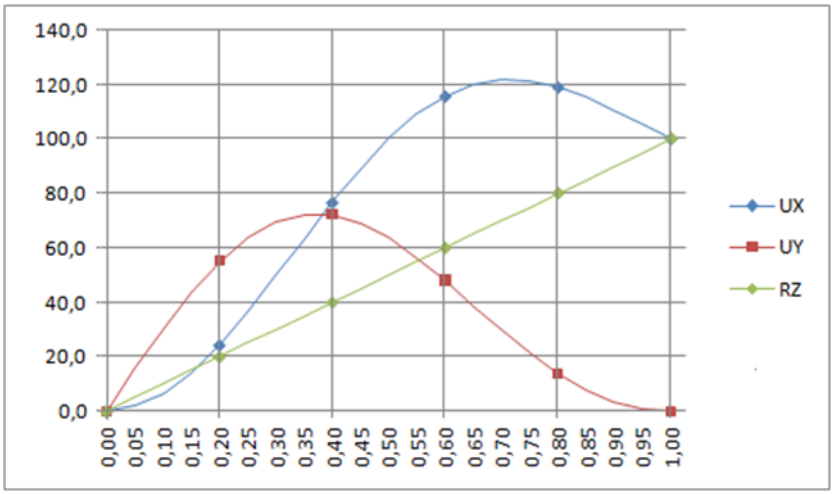



**DATA DEFINITION**:

| | |
|---|---|
| **Loading:** | Concentrated moment at the end point M = 3 436,1 [kN*m] (applied in 36 equal increments) |
| **Boundary Conditions:** | Built-in at the begin |
| **Material Properties:** | $E=210x10^9$ N/m$^2$ ; $\nu=0,0$ (density not considered) |
| **Element Type:** | 8 beam elements, 40 load increments, analysis P-delta, nonlinear. |
| **Data File:** | StaticAnalysis.Nonlinear.LargeDisplacements.NAFEMS_GNL5.rtd |

**RESULTS COMPARISON**:

The dimensionless value of tip displacements and rotations (UX=100*ux/L, UY=100*uy/L, RZ=100*rz/2*π) are presented on the analytical plot, and summarized in the following table.




| Normalized Horizontal Displacement at Tip Versus Normalized Bending Moment | | | |
|---|---|---|---|
| **M/Mmax** | **Analytical** | **Robot Structural** | **Difference** |
| 0.2 | 24.3 | 24.2 | 0.41% |
| 0.4 | 76.6 | 76.5 | 0.13% |
| 0.6 | 115.8 | 115.7 | 0.09% |
| 0.8 | 118.9 | 119.2 | 0.25% |
| 1 | 100 | 100 | 0.00% |

| Normalized Vertical Displacement at Tip Versus Normalized Bending Moment | | | |
|---|---|---|---|
| **M/Mmax** | **Analytical** | **Robot Structural** | **Difference** |
| 0.2 | 55 | 55.0 | 0.07% |
| 0.4 | 72 | 72.3 | 0.37% |
| 0.6 | 48 | 48.4 | 0.90% |
| 0.8 | 13.7 | 14.0 | 2.04% |
| 1 | 0 | 0.0 | 0.00% |

| Normalized Rotations at Tip Versus Normalized Bending Moment | | | |
|---|---|---|---|
| **M/Mmax** | **Analytical** | **Robot Structural** | **Difference** |
| 0 | 20 | 20.00 | 0.00% |
| 0 | 40 | 40.00 | 0.00% |
| 0 | 60 | 60.00 | 0.00% |
| 0 | 80 | 80.00 | 0.00% |

| | | | |
|---|---|---|---|
| 0 | 100 | 100.00 | 0.00% |

# Conclusions

The results and accuracy achieved in the verification examples confirm the quality and reliability of Robot Structural Analysis Professional. This state-of-the-art structural analysis and design software gives sufficient accuracy limited only by the precision of modeling.